probing the conformal window on the lattice

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XQCD 2013 – Bern, CH
probing the conformal window on the lattice (*with fundamental fermions*)

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Is there an IR fixed point in SU(3) Nf=12 theory?

Ishikawa, Iwasaki, Nakayama, Yoshie (phase structure, correlation fn.)

Appelquist, Fleming, Neil, M.Lin, Schaich (running coupling, mass spectrum)

Deuzeman, Lombardo, Pallante, Miura, da Silva (finite temperature)

Cheng, A. Hasenfratz, Petropoulos, Schaich (MCRG, phase structure, Dirac eigenmodes)

DeGrand (mass spectrum)

LatKMI (mass spectrum)

D.Lin, Ogawa, Ohki, Shintani (running coupling)

Fodor, Holland, Kuti, Nogradi, Schroeder, (running coupling, phase structure, spectrum)

Jin and Mawhinney (phase structure)

Slide adapted from E. Itou
our collaboration

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outline

• background

• the pre-conformal regime

• the conformal window
outline

• background

• the pre-conformal regime

• inside the conformal window
beta function and conformality

\[ \beta(g) = -b_0 g^3 - b_1 g^5 + (\ldots) \]

\[ b_0 = \frac{1}{16\pi^2} \left(11 - 2 \frac{N_f}{3}\right), \quad b_1 = \frac{1}{(16\pi^2)^2} \left(102 - 38 \frac{N_f}{3}\right) \]
the $T-N_f$ phase diagram

- Quark gluon plasma
- Hadronic phase
- End point Chiral Phase Boundary
- Conformal window
- AF Lost

$T_c$, $3$, $16.5$
the $N_f - 1/g$ phase diagram
outline

• background

• the pre-conformal regime

• the conformal window
hunting a thermal transition

Existence of True thermal transition

Absence of True thermal transition

$N_f < N_f^*$

$N_f > N_f^*$

$N_f$
a thermal transition with $N_f = 8$

$N_t = 6$

$N_t = 12$

opening the conformal window

\[ N_f = 6 \]

\[ N_f = 8 \]

J. Braun, H. Gies; JHEP 0606 024 (2006)
opening the conformal window

\[ N_f^C = 10.4 \pm 1.2 \]

Nuclear Physics B, 871 (2013)
opening the conformal window

- We would like to find a direct evidence of the singularity that opens the CW from \( \left( \frac{T_C}{M} \right)(N_f^*) = 0 \), where M is an UV scale that should disentangle the behavior in the IR.

- In the work above it was calculate at the plaquette scale with the help of the two-loop beta function.

- How can we define the UV scale directly from lattice measurements, without using perturbation theory?
work in progress: \( T_c / \sqrt{\sigma} \)

Quantity serves as input for model building in Gauge/Gravity duality models.
See, e.g., Gursoy et. al. arXiv:1006.5461

outline

• background

• the pre-conformal regime

• the conformal window
evidences of conformality

a tale of two jumps

SU(3) with $N_f = 12$
fundamental flavors

Tree level Symanzik
+ Naik improved action
a game of symmetries

\[ R = \frac{\partial \langle \bar{q}q \rangle}{\partial m} \frac{\langle \bar{q}q \rangle}{m} = \frac{\chi_\sigma}{\chi_\pi} \]
signatures of the exotic phase

\[ \text{Asym} \propto C(1 - (-1)^t)(e^{-mt} - e^{-m(T-t)}) \]
effect of improvement I

Second jump disappears with an unimproved action!
The Transfer matrix of a Symanzik improved lattice gauge action is no longer Hermitean. (Luscher, Weisz 1984).

The appearance of complex eigenvalues opens up the possibility of the emergence of new phases.

Where and how these phases appear will depend on the specifics of the improvement being used.
effect of improvement II

• In our specific case, the exotic phase will appear as a consequence of the competition between nearest and third nearest interactions introduced by Naik improvement.

• But this is a general feature of improved theories at strong coupling and such exotics may be observed by other groups using different actions (e.g. maybe the $S^4$ broken phase observed by A. Hasenfratz et al.).

• Since our studies on chiral symmetry restoration were carried at weaker couplings these results are not affected by this exotic.
order of the transition

gauge + fermion improved action

naive action
work in progress

- New ensembles with $N_f = 12, 16$ at volumes up to 32x32 are under study.

- Study of the eigenvalues and update on the study of the spectrum.
work in progress: mass dependence of eigenvalues

\[ N_f = 12, V = 32 \times 32, \beta = 3.900 \]

No zero modes!

\[ N_f = 16, V = 32 \times 32, \beta = 3.300 \]
Smearing pushes eigenvalues to higher values and suppresses chirality
mass ratios

**QCD:**

\[ m_\pi \propto \sqrt{m_q} \]

\[ m_\rho \propto m_0 + bm \]

\[ \frac{m_\pi}{m_\rho} \propto \sqrt{m_q} \]

**Conformal:**

\[ m_\pi \text{ & } m_\rho \propto m^\alpha \]

\[ \frac{m_\pi}{m_\rho} \approx 1 \]
the Edinburgh plot

* Our data

Heavy quark limit

QCD physical point

PRELIMINARY
conclusions and outlook

• Our results are consistent with an conformal window opening below $N_f = 12$.

• Improved lattice theories might develop exotic phases at strong coupling

• Final analysis on the way -> stay tuned!

• Thank you!
backup – CPB scaling

\[ k_{SB} \propto k_0 \theta(N_f^{cr} - N_f) |N_f^{cr} - N_f|^{-1/\Theta} \exp \left( - \frac{\pi}{2\epsilon \sqrt{\alpha |N_f^{cr} - N_f|}} \right) \]

power-law
(due to running coupling)

exponential-law
(Miransky-KBT scaling)

\[ \beta(g^2) = -\Theta(g^2 - g_0^2) + \ldots \quad \Theta < 0 \]
backup – bulk $N_f = 16$

SU(3) with $N_f = 16$, $am = 0.025$, $V = 16 \times 24$
backup – chi_U(1)
backup - improvement

• Consider the free lattice fermion propagator:

\[ S_F(p)^{-1} = \sum_{\mu} i\gamma_{\mu} \left( \frac{9}{8} \sin p_{\mu} - \frac{1}{24} \sin 3p_{\mu} \right) \]

• Contributions from the interacting theory can modify the coefficients of each sine term.

• In particular, a change in the sign of the second term will induce imaginary poles and ghosts and signal the emergence of the exotic phase.
backup - improvement

• Baryon number density will receive contributions from nearest and third nearest interactions:

\[ n(\mu) = \frac{d}{d\mu} \log Z(\mu) = n_1(\mu) + n_3(\mu) \]

• Baryon number conservation at \( \mu = 0 \) can be realized with \( n_1 = n_3 = 0 \) or with \( n_1 = -n_3 \neq 0 \).

• At strong coupling, when third nearest interactions become relevant, the second realization is possible. A non-zero value of \( n_1(\mu) \) allows for both an oscillation term in the PS correlator and the time asymmetry in all correlators.
backup I – eigenvalue distribution

Eigenvalues distribution, $N_f = 16$, $\beta = 3.300$, 100 first eigenvalues

- **$am = 0.01$**
- **$am = 0.02$**
backup II – cooling effect on eigenvalues
backup - relation to chiral condensate

\[ M_{TT}^2 \]

exact chiral symmetry anomalous dimensions \( \neq 0 \)

broken chiral symmetry

\[ \langle \bar{\psi} \psi \rangle^2 \]

PRELIMINARY