Thermodynamic properties of QCD in external magnetic fields

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xQCD, Bern, August 2013

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Magnetic fields and Quantum Chromodynamics

- early universe $\sqrt{eB} \simeq 2$ GeV

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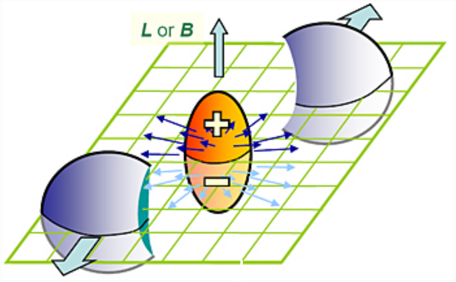
- heavy ion collisions (LHC) 0.1..0.5 GeV QCD scale!
  non-central collisions
  charged spectators
  B perp. to reaction plane



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Magnetic fields and Quantum Chromodynamics

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• neutron stars, magnetars	1 MeV	$B \simeq 10^{14}$ G
◦ cf. strongest field in lab		10^5 G (10^7 G unstable)
◦ refrigerator magnet		100 G
◦ earths magn. field		0.6 G

Magnetic fields and Quantum Chromodynamics

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- ◦ earths magn. field — 0.6 G

magn. fields as probes for our understanding of nonperturbative QCD

Technical details

idealized: constant external magn. field B + QCD in equilibrium

B as new axis of QCD phase diagram

lattice: abelian space-dependent phases on the links mimicking A_μ

B quantized and bounded, but no sign problem

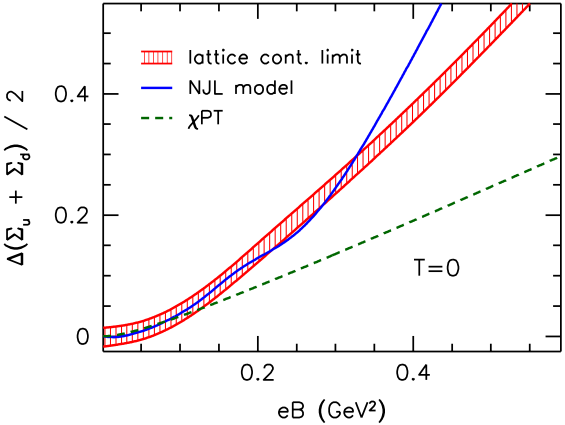
simulations similar to transition studies at $B = 0$ Budapest-Wuppertal

- tree-level improved gauge action
- stout smeared staggered fermions (rooting trick)
- 2 light quarks + strange quark, charges $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$
- physical pion masses
- B does not alter scale setting $a(\beta)$ and line of constant physics $m(\beta)$

state-of-the-art: $\sqrt{eB} = 0.1 \ldots 1$ GeV

Magnetic catalysis

- change of light quark condensate with B: Bali, FB et al. '12



$\Rightarrow$ condensate (dyn. mass) increased by magnetic field

- comp. to χPT, NJL Cohen, McGady, Werbos '07, Andersen '12; Gatto, Ruggieri '10
  well approximated unless $eB > 0.1, 0.3$ GeV^2 (approaches valid?)

Landau level picture

- free Dirac equation with magnetic field $B\vec{e}_z$ via say $A_y = Bx$:

$$-\not{D}^2 = -\partial_t^2 - \partial_z^2 - \partial_x^2 - (\partial_y + qBx)^2 + qB\,\sigma_{12} \qquad \sigma_{12} \propto [\gamma_1, \gamma_2]$$

plane waves   harm. oscillator   spin

$$\lambda^2 = p_t^2 + p_z^2 + |qB|(2n+1) + qB(2s)$$

Landau '30

$$p_t, p_z \in \mathbb{R} \qquad n = 0, 1, \ldots \qquad s = \pm 1/2$$

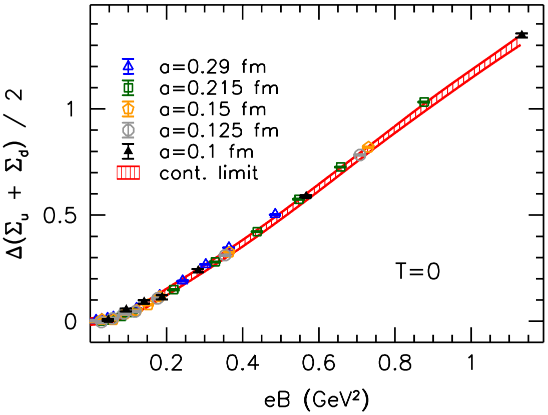
- lowest Landau level: $\lambda = 0$ (massless case)
  charged spin 0 (pions): $\lambda^2 = |eB| \Rightarrow$ mass grows: $\sqrt{m^2(0) + |eB|}$
  charged spin 1 (rho mesons): $\lambda^2 = -|eB| \Rightarrow$ mass reduces

- degeneracy of all Landau levels: $|\text{magn. flux}| = |qB| \cdot \text{area}$

- strong magnetic fields generate many small eigenvalues
  $\Rightarrow$ increase of condensate via $\rho(\lambda = 0)$ — Banks, Casher '80
  = 'Magnetic catalysis', robust in all models — Müller, Schramm^2 '92
  Gusynin, Miransky, Shovkovy '96

Magnetic catalysis (again)

- change of condensate with B at $T = 0$: Bali, FB et al. '12

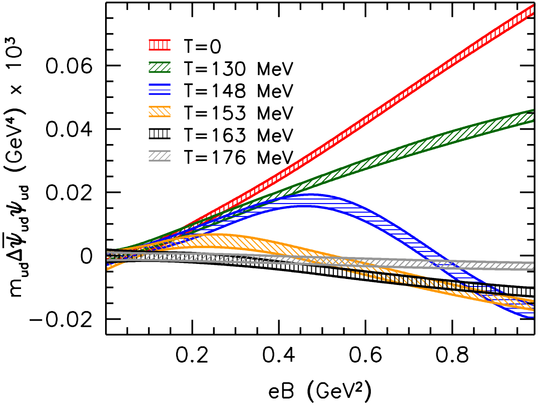


note that : $\Delta ... = ...(B) - ...(0)$ removes add. divergences (as would T)
$m \cdot ...$ removes mult. divergences

- prefers broken chiral symmetry $\overset{?}{\leadsto}$ higher crit. temperature

Inverse magnetic catalysis

- change of condensate with B at the QCD crossover: Bali, FB et al. '12



non-monotonic ⇒ magn. catalysis turns into inverse magn. catalysis

- phys. quark masses essential (as we have checked)

  higher masses in other lattice simulations D'Elia et al. 10, Ilgenfritz et al. 12

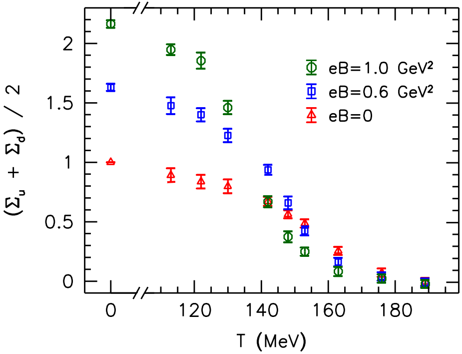
  IMC missed in almost all non-lattice approaches

Chiral restoration and deconfinement

- as a function of T at fixed B's:

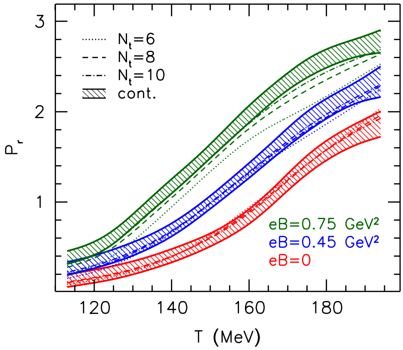
FB, Endrődi, Kovács '13

condensate



non-monotonic

Polyakov loop



monotonic

(pseudo-) T_c's from inflection points (for P harder to determine)

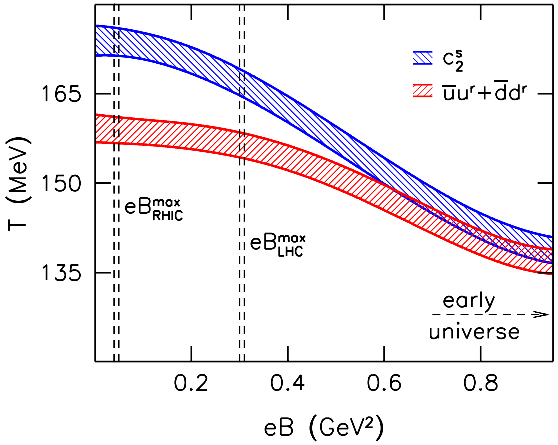
$\Rightarrow$ both (pseudo-)critical temperatures decrease

- no splitting between them

QCD phase diagram with magn. field

- (pseudo-) T_c as a function of magnetic field:

Bali, FB et al. '12



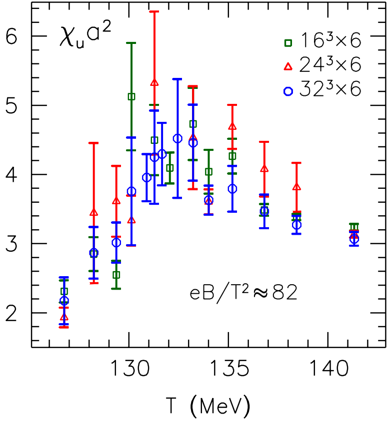
strange number susceptibility
light quark condensate
both renormalized

$\Rightarrow$ T_c decreases by O(10) MeV for $eB \lesssim 0.5\ \mathrm{GeV}^2$

phenomenologically relevant?? (cmp. our setting)

Nature of the transition

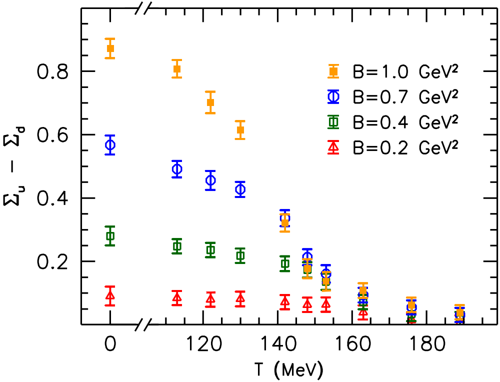
- volume dependence of light susceptibility:



no volume scaling

$\Rightarrow$ remains a crossover up to $\sqrt{eB} \simeq 1$ GeV

- change of light quark condensate difference $\langle \bar{u}u \rangle - \langle \bar{d}d \rangle$ as a function of T: Bali, FB et al. '12

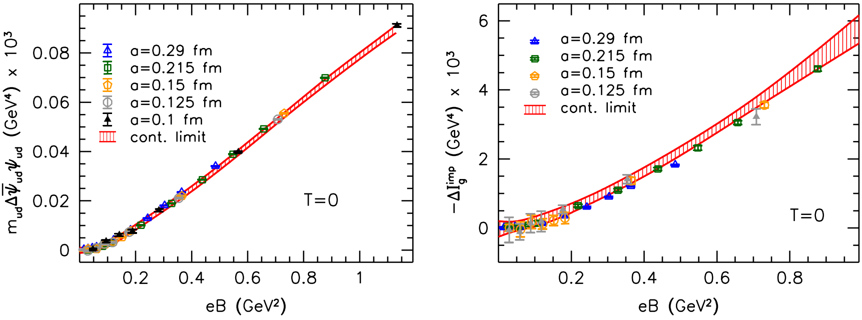


due to different el. charges $2e/3$ vs. $-e/3$ (mass degenerate)

$\Rightarrow$ order parameter, similar T-dependence as average condensate

Magnetic catalysis for gluons

- change of condensate and gluonic action: Bali, FB et al. '13



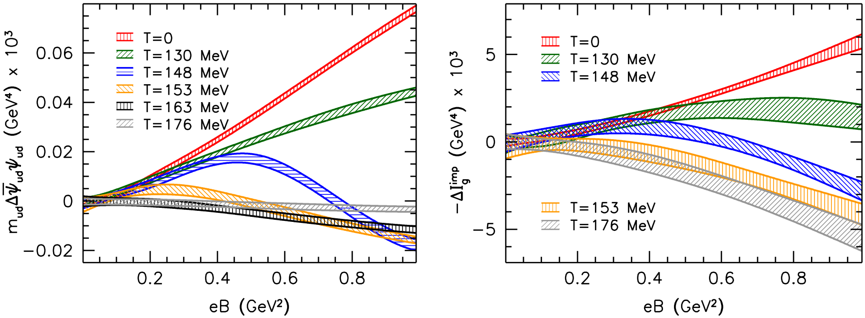
$\Rightarrow$ gluons inherit magnetic catalysis from quarks

since strongly coupled

magnitude $\mathcal{O}(100)$ larger for gluons, but $B = 0$ scale (= gluon condensate) already $\mathcal{O}(200)$ larger: relative effect larger on quarks

Inverse magnetic catalysis for gluons

- again change of condensate and gluonic action, now finite T:



non-monotonic behaviour, similar shape for quarks and gluons

⇒ gluons inherit inverse magnetic catalysis from quarks, too

connected in trace anomaly

Intermezzo: Trace anomaly

$$I = \epsilon - 3p \qquad \ldots \text{interaction measure, since free gas: } \epsilon = 3p$$

$$\stackrel{\text{lattice}}{=} -\frac{T}{V}\frac{d\log Z}{d\log a} \qquad \ldots \text{scale anomaly}$$

$$= -\frac{T}{V}\left(\frac{\partial \log Z}{\partial\beta}\frac{\partial\beta}{\partial\log a} + \frac{\partial\log Z}{\partial\log am}\frac{\partial\log am}{\partial\log a}\right) \qquad \beta = \frac{6}{g^2}$$

$$= -\left(\langle s_g\rangle\frac{-\partial\beta}{\partial\log a} + m\langle\bar{\psi}\psi\rangle\frac{\partial\log am}{\partial\log a}\right)$$

$$\Delta I = -\left(\Delta\langle s_g\rangle\underbrace{\frac{-\partial\beta}{\partial\log a}} + m\,\Delta\langle\bar{\psi}\psi\rangle\underbrace{\frac{\partial\log am}{\partial\log a}}\right)$$

change of gluonic action density and condensate multiplied by lattice beta and gamma function [LCP]

$\stackrel{!?}{\Rightarrow}$ similarity in B-dependence

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change of gluonic action density and condensate multiplied by
lattice beta and gamma function [LCP]

$\overset{!?}{\Rightarrow}$ similarity in B-dependence

Inverse magnetic catalysis: mechanism

$$\langle\bar\psi\psi\rangle^{\text{full}} = \frac{\int e^{-S_g}\det(\not{D}[B]+m)\,\text{tr}(\not{D}[B]+m)^{-1}}{\int e^{-S_g}\det(\not{D}[B]+m)}$$

$$\langle\bar\psi\psi\rangle^{\text{val}} = \frac{\int e^{-S_g}\det(\not{D}[0]+m)\,\text{tr}(\not{D}[B]+m)^{-1}}{\int e^{-S_g}\det(\not{D}[0]+m)} \quad B \text{ in observable}$$

$$\langle\bar\psi\psi\rangle^{\text{sea}} = \frac{\int e^{-S_g}\det(\not{D}[B]+m)\,\text{tr}(\not{D}[0]+m)^{-1}}{\int e^{-S_g}\det(\not{D}[B]+m)} \quad B \text{ in config. generation}$$

to lowest order approx. : $\langle\bar\psi\psi\rangle^{\text{full}} \simeq \langle\bar\psi\psi\rangle^{\text{val}} + \langle\bar\psi\psi\rangle^{\text{sea}}$ D'Elia, Negro '11

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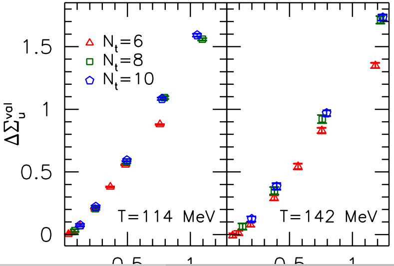
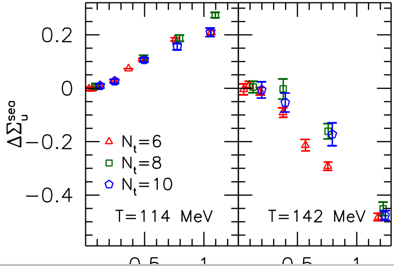
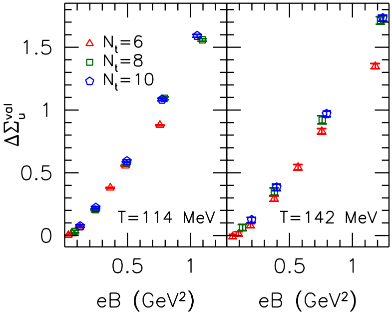
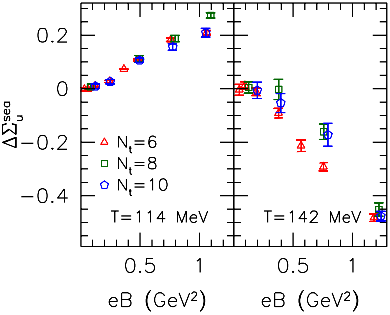
- $\Delta\langle\bar\psi\psi\rangle$ at low T and around T_c: FB, Endrődi, Kovács '13









$\not{D}$ has more small eigenvalues with B

in <u>valence</u> trace ⇓	⇓ in <u>sea</u> determinant
generates condensate	leads to a B-dep. probability
= statement about the change of the spectrum	= statement about the typical gauge field
(even quenched)	= feedback of quarks

- sea effect particularly effective near T_c: eff. potentials are flat
  why increasing at low T?

How to characterize change of gauge field?

Polyakov loop *P* increases with *B*

- *P* effectively changes quark bcs. away from antiperiodic

  deconfinement $\Rightarrow P \simeq 1 \Rightarrow$ large $\lambda(\not{D})$ 'Matsubara frequency'
  $\Rightarrow$ small density at zero $\Rightarrow$ small $\langle\bar{\psi}\psi\rangle \Rightarrow$ chiral symm. restoration

- effective action from fermion determinant in *P*-background:

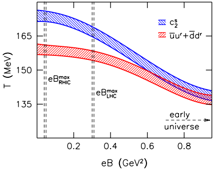
$$S^{\text{eff}}_{\text{free}} = -\log\det(\not{D}[B, P; T] + m) \quad \text{(free, no gluons)}$$

  prefers deconfining *P* the smaller the quark mass ✓
  (prefers confining *P* the larger imag. chemical potential ✓)
  prefers deconfining *P* the larger the magnetic field!

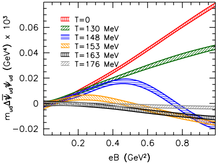
  washed out for heavy quarks

Summary so far

- phase diagram:
  - $T_c(B)$ decreases by O(10) MeV
  - still crossover



- magnetic catalysis: $\langle\bar{\psi}\psi\rangle(B) \nearrow$ at $T = 0$
  - robust: Landau level degeneracy
  - ⇕ also for gluons, cf. trace anomaly
- inverse magnetic catalysis: $\langle\bar{\psi}\psi\rangle(B) \searrow$ near T_c
  - quark back reaction: sea effect dominates
  - only at phys. masses
  - Polyakov loop $\nearrow$
  - important aspect for non-lattice approaches



Anisotropy I: Magnetic susceptibility

- tensor polarization: Ioffe, Smilga '84

$$\langle\bar{\psi}\sigma_{\mu\nu}\psi\rangle \propto F_{\mu\nu} \qquad \langle\bar{\psi}\sigma_{12}\psi\rangle \equiv qB\,\underbrace{\langle\bar{\psi}\psi\rangle\cdot\chi}_{\tau} + O((qB)^3)$$

  for radiative Ds meson transitions, anomalous magnetic moment of the muon, chiral-odd photon distribution amplitudes etc.

  $\chi = -2$ GeV^{-2} Bali, FB et al '12

  $\tau = -40$ MeV, at finite T like an order parameter

- contributes to free energy:

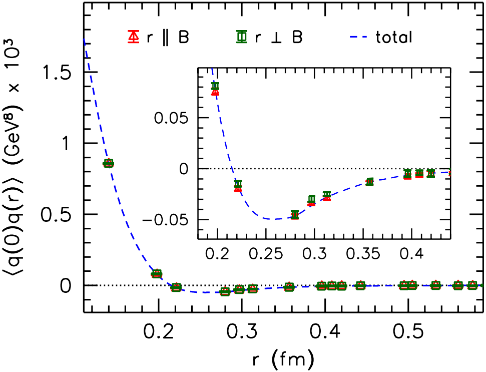
$$\frac{\partial F}{\partial B} \propto -\tau(qB) + \text{angular momentum} + O((qB)^3)$$

  but: total $O(B^2)$ at $T = 0$ completely fixed by charge renormalization

Anisotropy II: Topological charge

- two-point correlator in different directions:

$$\langle q(0)q(r)\rangle_{\vec{r}\,\|\,\vec{B}} \quad \text{vs.} \quad \langle q(0)q(r)\rangle_{\vec{r}\,\perp\,\vec{B}} \quad \text{vs.} \quad \langle q(0)q(r)\rangle$$



$\Rightarrow$ no stat. significant anisotropy

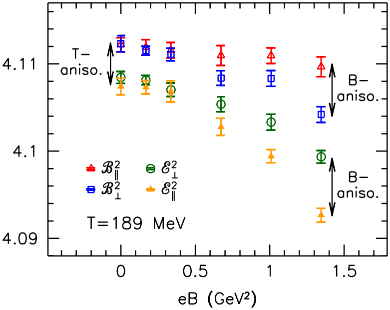
although topology related to quark zero modes (index theorem) and those should become elongated along B (Landau levels) …

Anisotropy III: Gluonic action

- field strength components (plaquettes) in various planes:

Bali, FB et al. '13

$$\langle \mathrm{tr}\, \mathcal{E}_\parallel^2 \rangle < \langle \mathrm{tr}\, \mathcal{E}_\perp^2 \rangle \overset{T \gtrsim T_c}{<} \langle \mathrm{tr}\, \mathcal{B}_\perp^2 \rangle < \langle \mathrm{tr}\, \mathcal{B}_\parallel^2 \rangle$$



same for coarse and heavy $N_f = 2$ simulations

Ilgenfritz et al. 12

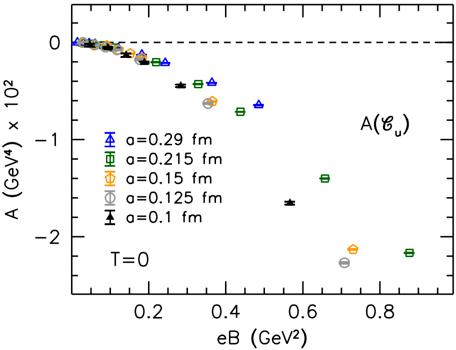
in line with perturbative Euler-Heisenberg eff. Lagrangian:

$$-\log\det(\not{D}[B,\mathcal{F};T=0]+m) \sim \frac{(qB)^2}{m^4}\left[\frac{5}{2}\langle \mathrm{tr}\, \mathcal{E}_\parallel^2\rangle - \langle \mathrm{tr}\, \mathcal{B}_\perp^2\rangle - \langle \mathrm{tr}\, \mathcal{E}_\perp^2\rangle - 3\langle \mathrm{tr}\, \mathcal{B}_\parallel^2\rangle\right]$$

Anisotropy IV: Quark action

- similarly:

$$\langle \bar{\psi} \not{D}_{\parallel} \psi \rangle > \langle \bar{\psi} \not{D}_{\perp} \psi \rangle$$



⇒ bigger than gluonic anisotropy

⇒ roughly independent of temperature

(An)isotropic pressure and magnetization

- pressure p is change of free energy under compression of volume, now sensitive to direction:

$$p_i = -\frac{L_i}{V}\frac{dF}{dL_i}$$

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- specify whether magn. field eB or magn. flux $\Phi = eB \cdot L_x L_y$ constant

phys. question, e.g. Φ-scheme: perfect conductors, field lines frozen in

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- homogeneous system: free energy extensive with density f

$$F = L_x L_y L_z \cdot f(eB) = L_x L_y L_z \cdot f(\frac{\Phi}{L_x L_y})$$

additional $L_{x,y} = L_\perp$-dependence $\Rightarrow$

$$\underbrace{p_\perp^{(B)} = p_\parallel^{(B)}}_{\text{isotropic pressure}} \; = \; \underbrace{p_\parallel^{(\Phi)} \neq p_\perp^{(\Phi)}}_{\text{anisotropic pressure}}$$

not very clear in the literature (!) on the lattice fortunately ...

…we've got a quantized magn. flux and thus the Φ-scheme, moreover:

$$p_\perp^{(\Phi)} - p_\parallel^{(\Phi)} = -\frac{\partial f}{\partial eB} \cdot L_\perp \frac{\partial eB}{\partial L_\perp}\bigg|_{\Phi=\text{const.}:\ eB \propto L_\perp} = -M \cdot eB$$

$\Rightarrow$ measure the magnetization M

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$$= -M \cdot eB$$

⇒ measure the magnetization M

anisotropic lattices mimicking different L_μ:

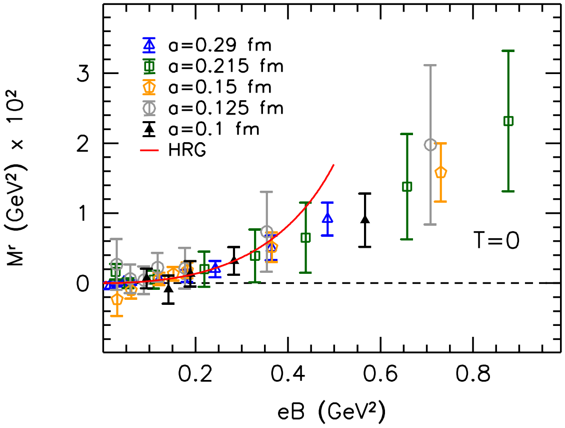
$$-M \cdot eB = -\zeta_g[A(\mathcal{E}) - A(\mathcal{B})] - \zeta_f \sum_f A_f$$

in terms of aforementioned anisotropies $A(\mathcal{F}) = \langle \text{tr}\, \mathcal{F}_\perp^2 \rangle - \langle \text{tr}\, \mathcal{F}_\parallel^2 \rangle$ and $A_f = \langle \bar{\psi} \not{D}_\perp \psi \rangle - \langle \bar{\psi} \not{D}_\parallel \psi \rangle$, Karsch coefficients:

$$\zeta_{g,f} \xrightarrow{\text{pert.}} 1 + \mathcal{O}(g^2)$$

first approx.: $\zeta_{g,f} = 1$

- after subtraction of $O(B^2)$ term (charge renormalization):



including result from Hadron Resonance Gas

Endrődi ’13

⇒ QCD vacuum is paramagnetic

similar results through integral methods:

$$f = \int_{\infty}^{m_{\text{phys.}}} dm \, \frac{\partial f}{\partial m} - f(m = \infty) \ldots \text{condensates}$$

Bali, FB et al. in prep.

$$= \int_{0}^{B} dB \, \frac{\partial f}{\partial B} + f(B = 0) \ldots \text{auxiliary magnetization}$$

Bonati et al. '13

Summary II

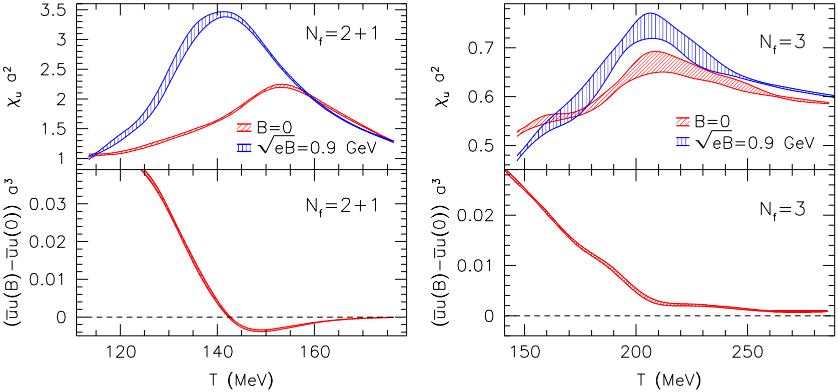
- anisotropies:
  - magn. susceptibility: $\langle\bar{\psi}\sigma_{\mu\nu}\psi\rangle \propto B$
  - field strength: $\langle \text{tr}\, \mathcal{E}_{\parallel}^2\rangle < \langle \text{tr}\, \mathcal{E}_{\perp}^2\rangle < \langle \text{tr}\, \mathcal{B}_{\perp}^2\rangle < \langle \text{tr}\, \mathcal{B}_{\parallel}^2\rangle$
    cf. perturbative Euler-Heisenberg
  - quark action: $\langle\bar{\psi}_f \not{D}_{\parallel}\psi_f\rangle > \langle\bar{\psi}_f \not{D}_{\perp}\psi_f\rangle$, dominates
  - topology: no anisotropy in correlator
- (an)isotropic pressure:
  - depends on scheme: fixed field vs. fixed flux
  - allows to compute on the lattice magnetization from anisotropies:
    QCD vacuum is paramagnetic
    effect on elliptic flow??

Backup: more simulation details

- tree-level improved gauge action
- stout smeared staggered fermions (rooting trick)
- 2 light quarks + strange quark, charges $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$
- lattice spacing set at $T = 0, B = 0$
  physical pion masses
  set by f_K, f_K/m_π and f_K/m_K
- $T = 0$: $24^3 \times 32$, $32^3 \times 48$ and $40^3 \times 48$ lattices
- $T > 0$: $N_t = 6, 8, 10$ meaning $a = 0.2, 0.15, 0.12$ fm
  $N_s = 16, 24, 32$ for finite volumes
- condensates from stochastic estimator method with 40 vectors
- magn. flux quanta: $N_B \leq 70 < \frac{N_x N_y}{4} = 144$

Backup: Mass sensitivity

- what if we put $(m_{\text{light}}, m_{\text{light}}, m_{\text{strange}}) \to (m_{\text{strange}}, m_{\text{strange}}, m_{\text{strange}})$?



T-dep. of *u*-susceptibility (top) and change of *u*-condensate (bottom)

$\Rightarrow$ effects of decreasing T_c & inverse magn. catalysis disappear

light quark masses are important

Backup: Physical values

- at $T = 0$ and $\overline{\text{MS}}$ scheme at 2 GeV:

$$\begin{aligned}\tau_{\text{up}} &= -(40.7 \pm 1.3)\text{ MeV}\\ \tau_{\text{down}} &= -(39.4 \pm 1.4)\text{ MeV}\\ \tau_{\text{strange}} &= -(53.0 \pm 7.2)\text{ MeV}\\ \chi_{\text{up}} &= -(2.08 \pm 0.08)\text{ GeV}^{-2}\\ \chi_{\text{down}} &= -(2.02 \pm 0.09)\text{ GeV}^{-2}\end{aligned}$$

quenched unrenorm.: $\tau_{\text{up/down}} = -52$ MeV — Braguta, Buividovich et al. 10

QCD sum rules: $\chi_{\text{light}} = -(2.11 \pm 0.23)\text{ GeV}^{-2}$ — Ball, Braun, Kivel 03

vector dominance: $\chi_{\text{light}} = -\frac{2}{m_\rho^2} \approx -3.3\text{ GeV}^{-2}$

- at finite T: $|\tau_{\text{light}}|$ decreases like an order parameter

inflection point: $T_c = 162(3)(3)$ MeV (compatible with $T_c^{\langle\bar\psi\psi\rangle}$ at $B = 0$)