The mass of the two lightest quarks

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XQCD

Workshop on QCD under extreme conditions

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Standard Model at low energies

- Low energies ($E \ll M_{\mathrm{W}}$): weak interaction is frozen
  only generates tiny effects, visible in the finite lifetime of the particles, e.g.

$\Rightarrow$ Standard Model reduces to QCD + QED

Precision theory for cold matter ($T \ll M_{\mathrm{W}}$) size and structure of atoms, solids, etc.

- QED is infrared stable

$\Rightarrow$ at low energies, electromagnetic interaction can be treated as a perturbation

- Parameters in Lagrangian: $g, \theta, e, m_u, m_d, m_s, m_c, m_b, m_t, m_e, m_\mu, m_\tau$

  Bohr radius: $$a = \frac{4\pi}{e^2\, m_e}$$

- Pattern of quark and lepton masses looks bizarre . . .
- This talk: $\boxed{m_u, m_d}$

Symmetries

- Symmetry plays an essential role in our understanding of nature at low energies
- QCD with N_f massless quarks: Hamiltonian has an exact chiral symmetry, SU$(N_f)_L \times$SU$(N_f)_R$
- Unless N_f is taken too large, $|0\rangle$ is symmetric only under the subgroup SU$(N_f)_{L+R}$ Symmetry is hidden, "spontaneously broken"

$\Rightarrow$ Spectrum contains $N_f^2 - 1$ Goldstone bosons

- m_u, m_d, m_s happen to be small

$\Rightarrow$ SU(3)$_L \times$SU(3)$_R$ is an approximate symmetry of QCD

- broken spontaneously: $|0\rangle$ not invariant
- broken explicitly: $\mathcal{L}_{\text{QCD}}$ not invariant
  Symmetry broken by mass term $m_u\,\bar{u}u + m_d\,\bar{d}d + m_s\,\bar{s}s$,
  but since m_u, m_d, m_s are small, the breaking is weak

Light quark masses as perturbations

- Masses of the light quarks enter the Hamiltonian via

$$H_{\text{QCD}} = H_0 + H_1$$
$$H_1 = \int d^3x \, \{m_u \, \bar{u}u + m_d \, \bar{d}d + m_s \, \bar{s}s\}$$

  H_0 describes u, d, s as massless, c, b, t as massive

  H_0 is invariant under SU(3)$_L \times$SU(3)$_R$

$$\boxed{\text{Expansion in powers of } m_u, m_d, m_s} \iff \boxed{\text{Perturbation series in powers of } H_1}$$

- H_0 treats π, K, η as massless particles

  H_1 gives them a mass

Gell-Mann-Oakes-Renner formula

- First order perturbation theory yields:

$$M_\pi^2 = (m_u + m_d) \times |\langle 0|\, \bar{u}u\, |0\rangle| \times \frac{1}{F_\pi^2}$$

$$\Uparrow \qquad\qquad\qquad \Uparrow$$

explicit spontaneous

Gell-Mann, Oakes & Renner 1968

Coefficient: decay constant F_π

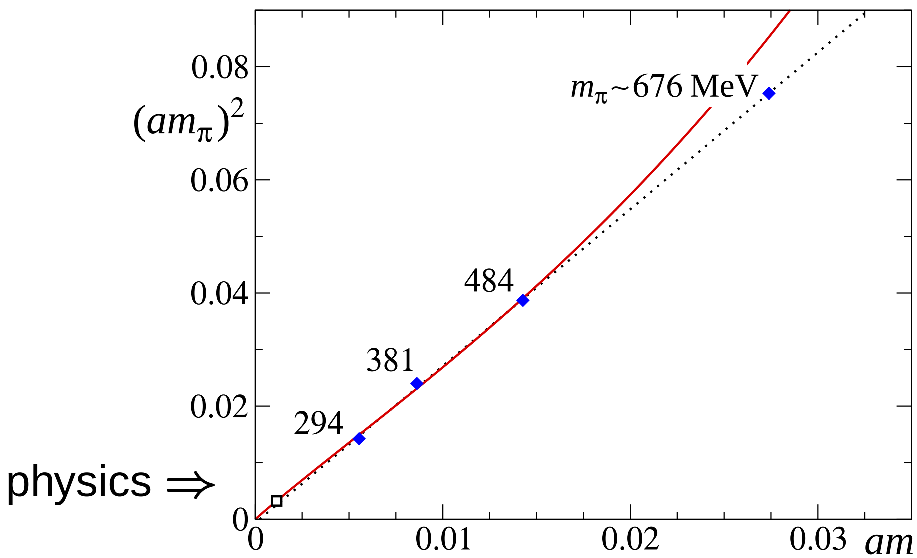
$$\langle 0|\, \bar{d}\gamma^\mu \gamma_5 u|\pi^+\rangle = i\, p^\mu \sqrt{2}\, F_\pi$$

Value of F_π is known from $\pi^+ \to \mu^+ \nu$

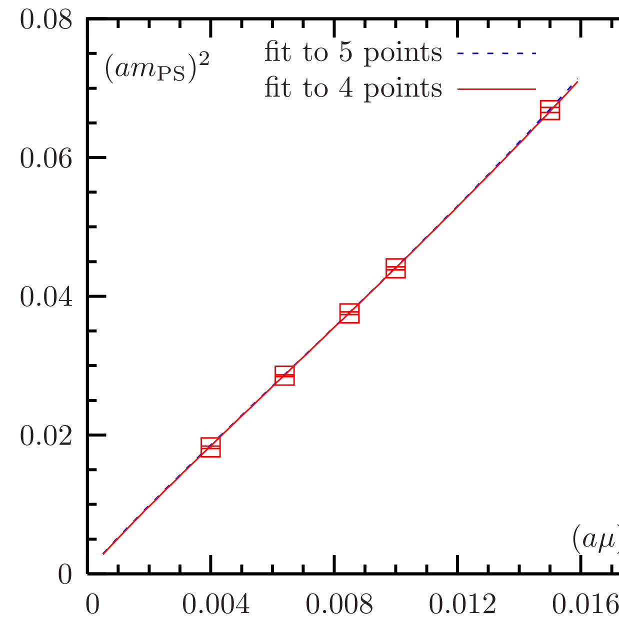
$\Rightarrow$ The main low energy properties of QCD can be understood on the basis of this formula

Lattice results for M_π

- GMOR formula is beautifully confirmed on the lattice: determine M_π as a function of $m_u = m_d = m$



Lüscher, Lattice conference 2005



ETM collaboration, hep-lat/0701012

- Proportionality of M_π^2 to m holds out to $m \simeq 10\times$ physical value of m_{ud}

Pattern of lowest levels

- $M_\pi^2 = (m_u + m_d)\, B + O(m^2)$

$\Rightarrow$ The energy gap of QCD is small because m_u, m_d happen to be small

- $M_{K^+}^2 = (m_u + m_s)\, B + O(m^2)$
  $M_{K^0}^2 = (m_d + m_s)\, B + O(m^2)$

$\Rightarrow$ M_K^2 is much larger than M_π^2, because m_s happens to be large compared to m_u, m_d

- Goldstone boson masses measure the strength of symmetry breaking

$\Rightarrow$ strongly violate SU(3)

- Check: first order perturbation theory also yields
  $M_\eta^2 = \frac{1}{3}\,(m_u + m_d + 4m_s)B + O(m^2)$

$\Rightarrow$ $M_\pi^2 - 4M_K^2 + 3M_\eta^2 = O(m^2)$

Gell-Mann-Okubo formula for M^2 $\surd$

Interface between the lattice approach and χPT

- The χPT formulae for the expansion of many quantities of physical interest in powers of m_u, m_d, m_s have been worked out to NNLO, not only masses and decay constants, also form factors, $\eta \to 3\pi$, ...
- m_s is not very small, terms of order m_s^2 yield sizable corrections.

  1. Pion physics: expansion in powers of m_u, m_d works very well.
  2. SU(3) is a decent approximate symmetry: symmetry breaking parameter $m_s - m_{ud}$ must be small, meaningful to expand in powers of $m_s - m_{ud}$.

  - The two properties can be understood if the expansion in powers of m_u, m_d, m_s makes sense at the physical values of the quark masses.
  - I do not know of an alternative explanation.
- Need to understand why some of the collaborations find that the χPT formulae do not describe their data on the quark mass dependence of M_π, M_K, F_π, F_K well.
- The constants relevant at NNLO are still poorly known. Often, theoretical estimates are used, obtained by saturating sum rules with resonance contributions. Those constants that govern the dependence on the quark masses, however, represent integrals over scalar spectral functions. Scalar meson dominance does not work!

$\Rightarrow$ Theoretical estimates can at best indicate the order of magnitude.
- The lattice approach is the ideal method for the determination of the LECs !

Zweig rule

- Of particular interest: understanding the Zweig (Okubo-Zweig-Iizuka) rule.

$$\{F, B, \Sigma\} = \left\{F_\pi, \frac{M_\pi^2}{m_u + m_d}, |\langle 0|\, \bar{u}u\, |0\rangle|\right\}_{m_u, m_d \to 0}$$

  If the Zweig rule was exact, these quantities would be independent of m_s.
- Low energy theorem (exact, for any value of m_s): $\Sigma = F^2 B$.
- Can determine the Zweig rule violations without reference to χPT: compare values at $m_s =$ physical and at $m_s = 0$: $F/F_0, B/B_0, \Sigma/\Sigma_0$.
- At NLO in the expansion in powers of m_s, the violations of the Zweig rule are described by the LECs L_4, L_6 (in the large N_c limit, these constants vanish).
- Only two papers without red tags in FLAG review: MILC (2009), HPQCD (2013). Inserting the values quoted for L_4, L_6 in the NLO formulae of χPT, I get

	F/F_0	B/B_0	Σ/Σ_0
MILC (2009)	1.12(4)	1.10(7)	1.34(13)
HPQCD (2013)	1.10(8)	1.12(8)	1.32(28)
GL (1985)	1.0(1)	1.0(2)	1.0(3)

$\Rightarrow$ Evidence for small Zweig rule violations, consistent with the crude old estimates. Paramagnetic inequalities of Descotes-Genon, Girlanda & Stern are confirmed.

Zweig rule

first order in m_s	F/F_0	B/B_0	Σ/Σ_0
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The Zweig rule violations roughly amount to a common change in scale:

$$F \simeq Z F_0, \quad B \simeq Z B_0 \quad \Rightarrow \quad \Sigma \simeq Z^3 \Sigma_0 \quad \text{with} \quad Z \simeq 1.10(5)$$

- MILC has evaluated the ratios to all orders in m_s:

all orders	F/F_0	B/B_0	Σ/Σ_0
MILC (2009)	1.10(4)	1.20(7)	1.48(16)

For F/F_0, the corrections are small, but for B/B_0, the central values of the terms of order m_s and m_s^2 (or higher) are of the same size ...
The Zweig rule deserves more attention !

- HPQCD instead evaluated the quark condensates at the physical quark masses:

$$\frac{\langle 0|\bar{s}s\,|0\rangle}{\langle 0|\bar{u}u\,|0\rangle} = 1.08(16)(1)$$

Confirms that SU(3) is a decent approximate symmetry: the symmetry breaking generated by $m_s - m_{ud}$ is too small to stick out from the noise of the calculation.

Isospin breaking

- The symmetry properties of the vacuum shield the pions from isospin breaking.
  - The difference between m_u and m_d only generates a tiny effect of order $M^2_{\pi^+} - M^2_{\pi^0} \propto (m_u - m_d)^2$.
  - The mass difference between π^0 and π^+ is due almost exclusively to electromagnetism.

  $\Rightarrow$ More easy to determine the mean mass $m_{ud} \equiv \frac{1}{2}(m_u + m_d)$ than the difference $m_u - m_d$.
- Estimate the e.m. self-energies with the Dashen theorem:

$$M^2_{K^+}\Big|_{e.m.} = M^2_{\pi^+}\Big|_{e.m} \qquad\qquad M^2_{\pi^0}\Big|_{e.m.} = M^2_{K^0}\Big|_{e.m.} = 0$$

Quark mass ratios

- Solve the tree level mass formulae for the ratios m_s/m_{ud} and m_u/m_d:

$$\frac{m_s}{m_{ud}} = \frac{M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+}}{M^2_{\pi^0}} = 25.9$$

Weinberg 1977

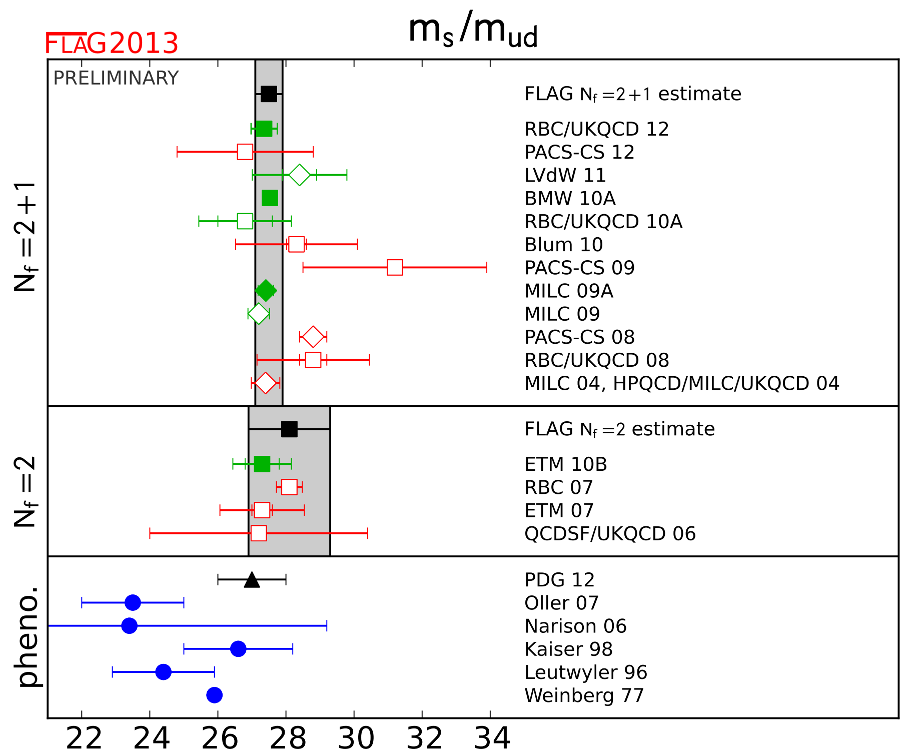
$$\frac{m_u}{m_d} = \frac{M^2_{K^+} - M^2_{K^0} + 2M^2_{\pi^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 0.56$$

- Low energy theorems, valid to leading order of the chiral expansion.
  Corrections from higher orders ? Could they strongly modify the numerical result ?

  What is the uncertainty to be attached to these predictions ?

Lattice

- Lattice results for $\dfrac{m_s}{m_{ud}}$:



preliminary lattice average quoted by FLAG:

$$\boxed{\frac{m_s}{m_{ud}} = 27.5 \pm 0.4}$$

accuracy reached: 1.5 %

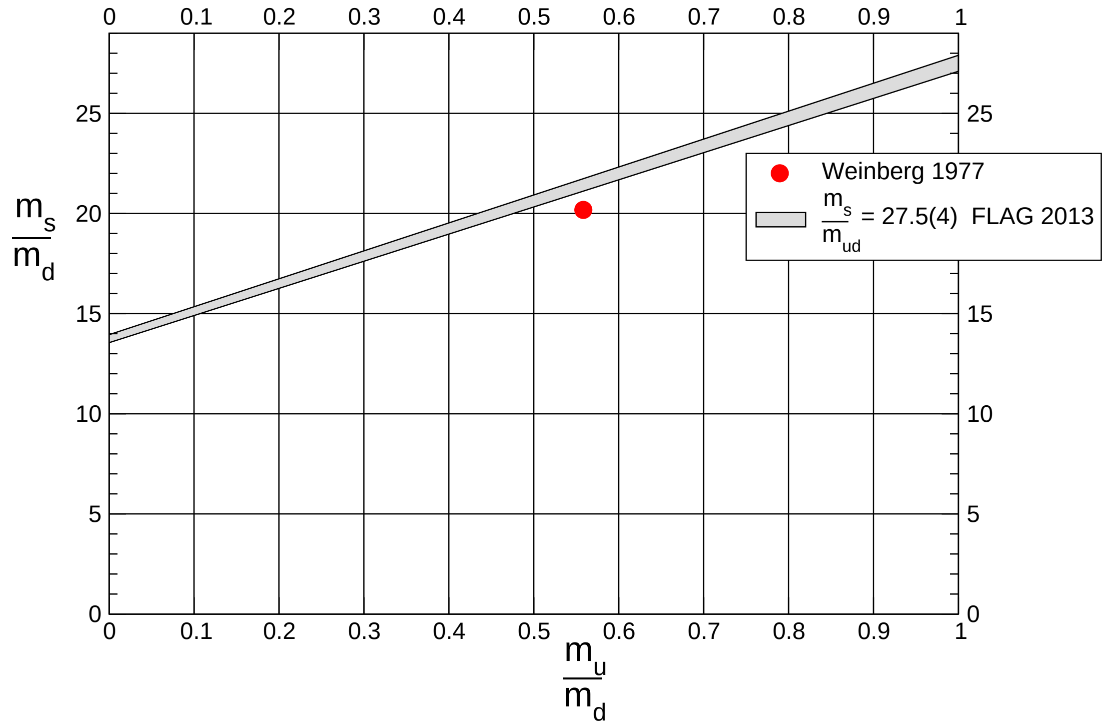
27.5 = 25.9 + 1.6

$\Uparrow$

higher orders

$\Rightarrow$ correction is small, leading term of chiral perturbation series dominates

Lattice



- Most lattice calculations are done in pure QCD.
- For m_s/m_{ud}, this is a good approxiation, because the uncertainties in the violations of the Dashen theorem do not strongly affect this ratio.
- For m_u/m_d, the situation is different. Lattice simulations of QCD + QED cannot be done with the same level of confidence as for QCD alone: not all systematic errors are under control (quenched photons, finite size effects for interactions of long range).

Low energy theorem valid to NLO

- The lattice result for m_s/m_{ud} determines the size of the correction in the relation

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + m_{ud}}{m_u + m_d}\left\{1 + \Delta_M\right\}$$

$$m_s/m_{ud} = 27.5 \pm 0.4 \quad \Rightarrow \quad \Delta_M = -0.057 \pm 0.013.$$

- Remarkably, chiral symmetry implies that the correction of NLO in the ratio of mass splittings is the same:

$$\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} = \frac{m_d - m_u}{m_s - m_{ud}}\left\{1 + \Delta_M + O(\mathcal{M}^2)\right\}$$

Hence the quark mass ratio

$$Q^2 \equiv \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}$$

is given by a ratio of meson masses, up to corrections of NNLO:

$$Q^2 = \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2}\cdot\frac{M_K^2}{M_\pi^2}\left\{1 + O(\mathcal{M}^2)\right\}$$

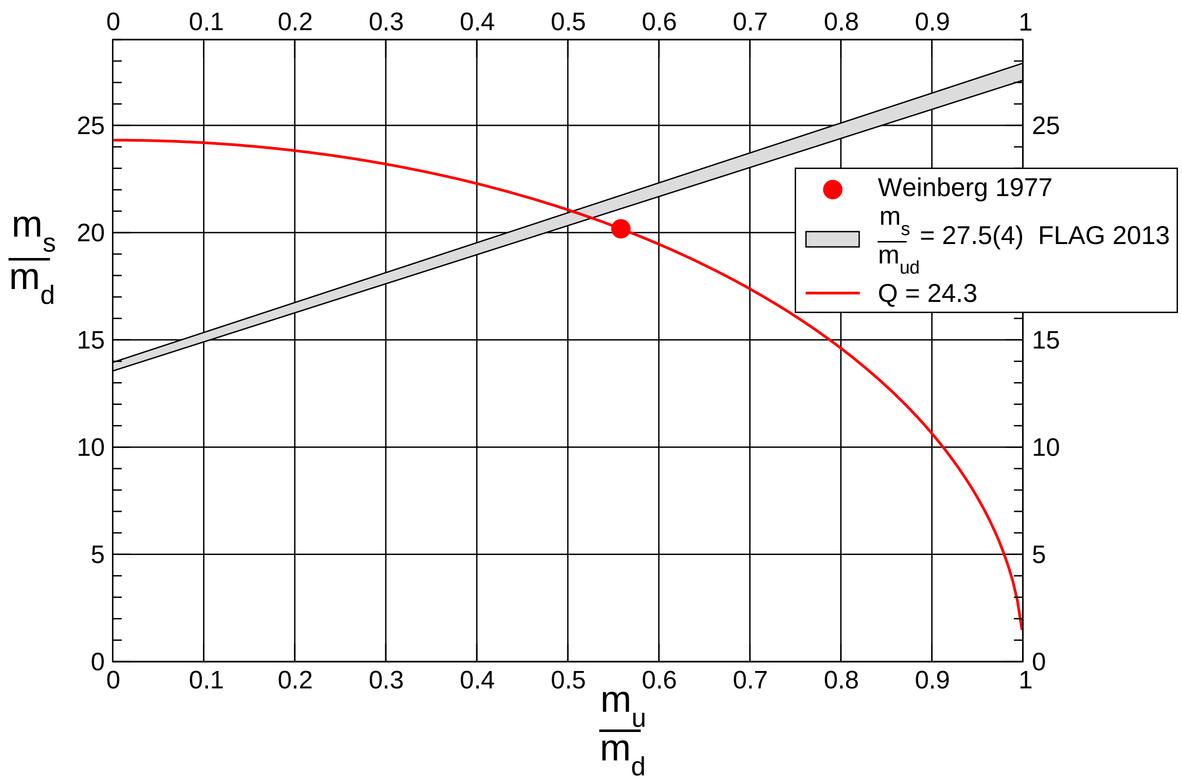
Gasser & L. 1985

Consequences of the low energy theorem for Q

- Insert Weinberg's leading order ratios $\Rightarrow$ Q = 24.3.
- Q^2 is a ratio of quark mass squares

$\Rightarrow$ a given value of Q imposes a homogeneous quadratic constraint on m_u, m_d, m_s

$\Rightarrow$ represents an ellipse in the plane of the quark mass ratios:



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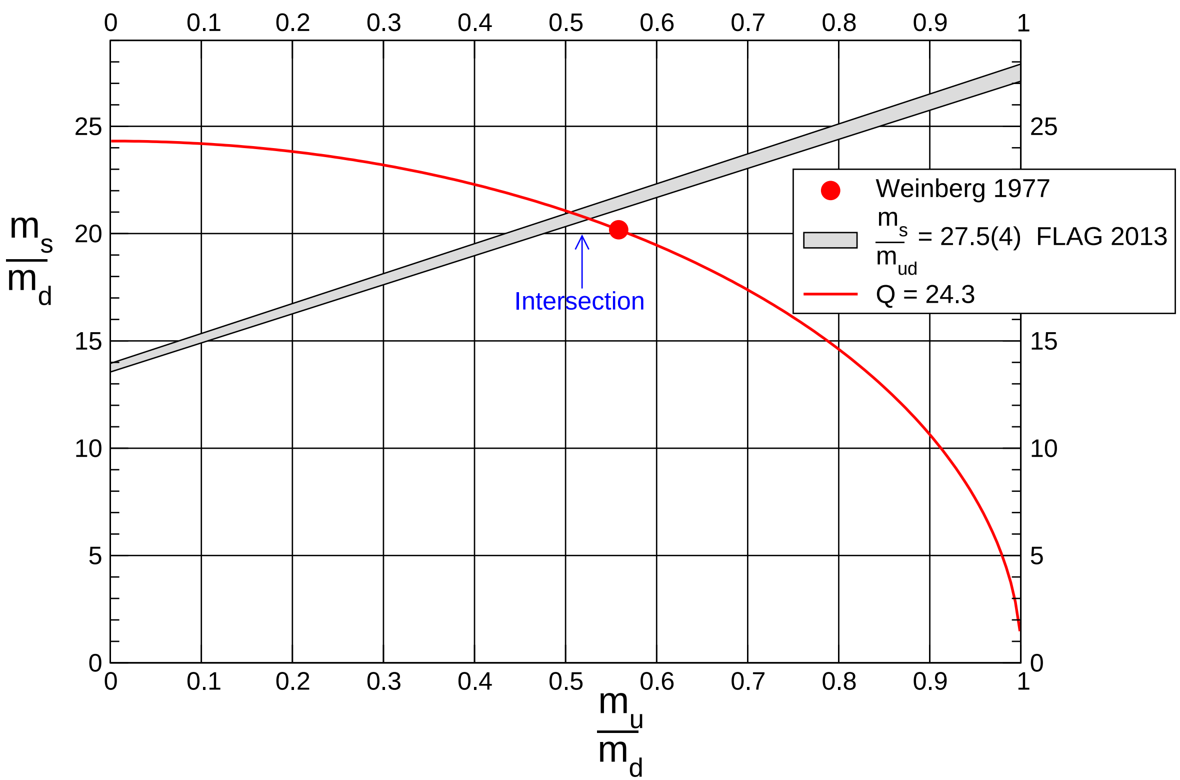


- Critical input here is the "Dashen theorem": Weinberg's estimates for the quark mass ratios account for QED only to LO.

$\eta \to 3\pi$

- The decay $\eta \to 3\pi$ provides a better handle on Q than the mass splitting between K^+ and K^0, because the e.m. interaction is suppressed (Sutherland's theorem).
  - For $e = 0$ and $m_u = m_d$, isospin is conserved, hence G-parity is conserved. In this limit, the η is a stable particle: $G_\eta = 1$, $G_\pi = -1$.
  - $\Rightarrow$ Since the e.m. contributions are tiny, the transition amplitude is to a very good approximation proportional to $(m_u - m_d)$.
  - Parameter free prediction for the leading term of the chiral perturbation series:

$$A(\eta \to \pi^+\pi^-\pi^0) = -\frac{\sqrt{3}}{4} \cdot \frac{m_d - m_u}{m_s - m_{ud}} \cdot \frac{s - \frac{4}{3}M_\pi^2}{F_\pi^2}$$

  - Compare leading term in the chiral expansion of the $\pi\pi$ scattering amplitude:

$$A(\pi\pi \to \pi\pi) = \frac{s - M_\pi^2}{F_\pi^2}$$

- In both cases, the leading term is linear in s and contains an Adler zero

  $s_A = M_\pi^2$ for $\pi\pi$ scattering $\qquad$ $s_A = \frac{4}{3}M_\pi^2$ for η decay
- The analytic structure of the two amplitudes is very similar.
- In both cases, the higher order contributions of the chiral perturbation series are dominated by the final state interaction among the pions.

One loop

- Most remarkable property of the one loop representation: expressed in terms of F_π, M_π, M_K, M_η, Q, all LECs except L_3 drop out. Gasser & L. 1985

$$A(\eta \to \pi^+\pi^-\pi^0) = -\frac{1}{Q^2} \cdot \frac{M_K^2(M_K^2 - M_\pi^2)}{3\sqrt{3}M_\pi^2 F_\pi^2} \cdot M(s,t,u)$$

- Moreover, L_3 concerns the momentum dependence of the amplitude, can be determined quite well from $\pi\pi$ scattering.

$\Rightarrow$ At one loop, the result for the rate is of the form

$$\Gamma_{\eta\to\pi^+\pi^-\pi^0} = \frac{C}{Q^4} \qquad\qquad Q^2 \equiv \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}$$

where C is a known constant $\Rightarrow$ Q can be determined from the observed rate.

- The main problem is not the uncertainty in L_3, but the contributions from higher orders. In 1985, we estimated the uncertainty in the result for Q at

$$\frac{1}{Q^2} = (1.9 \pm 0.3)\cdot 10^{-3} \qquad \leftrightarrow \qquad Q = 22.9^{+2.1}_{-1.6}$$

Gasser & L. 1985

- The result is consistent with the value $Q = 24.3$ obtained from the kaon mass difference with the Dashen theorem, but the uncertainties are large.

Dispersion theory

- The structure of the decay amplitude is governed by the final state interaction. Standard method for the analysis of this interaction: dispersion theory.
- Up to and including NNLO, the amplitude can be represented in terms of 3 functions of a single variable: Fuchs, Sazdjian and Stern 1993

$$M(s,t,u) = M_0(s)+(s-u)M_1(t)+(s-t)M_1(u)+M_2(t)+M_2(u)-\tfrac{2}{3}M_2(s)$$

(discontinuities from partial waves with $\ell \geq 2$ start contributing only at N^{3}LO).

- The dispersion relations obeyed by the three functions can be brought to the form

$$M_I(s) = \Omega_I(s)\left\{P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty} ds' \,\frac{\sin\delta_I(s')\hat{M}_I(s')}{|\Omega_I(s')|s'^n(s'-s)}\right\} \qquad I = 0,1,2$$

where $\delta_0(s), \delta_1(s), \delta_2(s)$ are the S- and P-wave phase shifts of $\pi\pi$ scattering and

$$\Omega_I(s) \equiv \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \,\frac{\delta_I(s')}{s'(s'-s)}\right\}$$

is the corresponding Omnès factor.

Anisovich & L. 1996

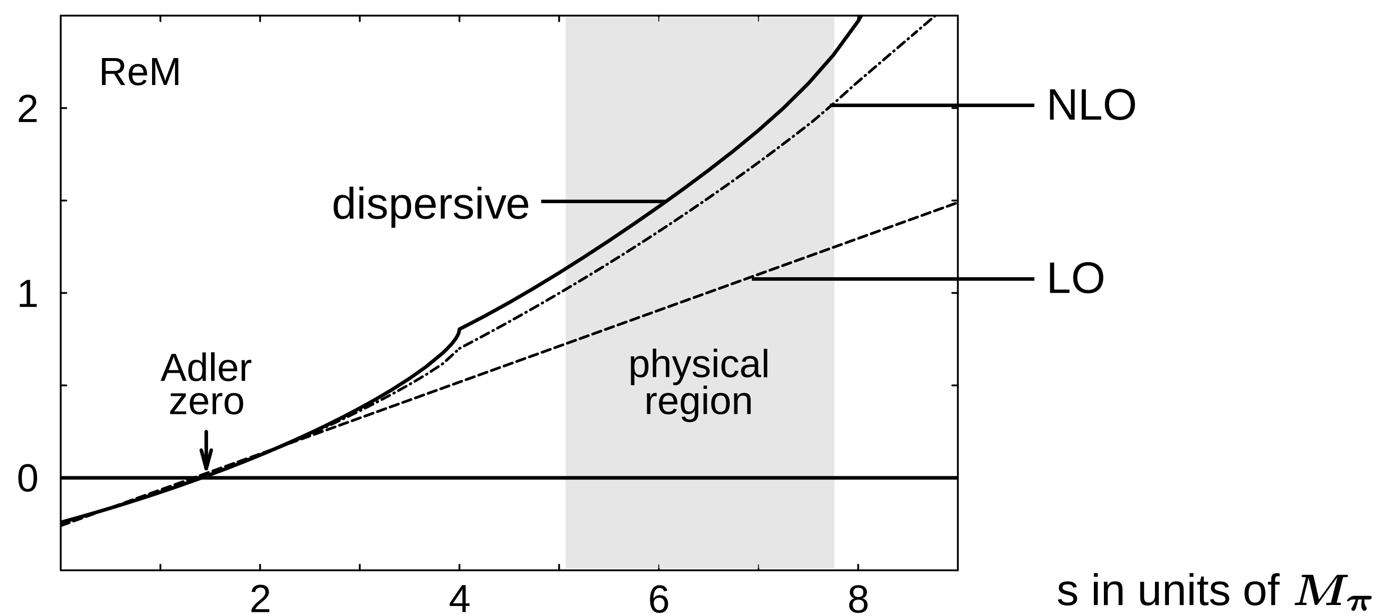
The polynomials $P_0(s), P_1(s), P_2(s)$ collect the subtraction constants.

$\Rightarrow$ S- and P-wave phase shifts of $\pi\pi$ scattering are needed. If these are known, dispersion theory fixes the amplitude up to the subtraction constants.

Dispersive analysis of η decay

- Main difference to $\pi\pi$ scattering: the subtraction constants relevant for $\eta \to 3\pi$ cannot be predicted to the same precision.
  - Can analyze $\pi\pi$ scattering by treating only m_u and m_d as small: SU(2)×SU(2)
  - In η decay, need to treat m_s as an expansion parameter as well: SU(3)×SU(3)
  - Only the occurrence of an Adler zero follows from SU(2)×SU(2) symmetry alone.
- The subtraction constants can be estimated by comparing the dispersive and chiral representations at small values of s, t or u and requiring the occurrence of an Adler zero at the proper place.

Anisovich & L. 1996



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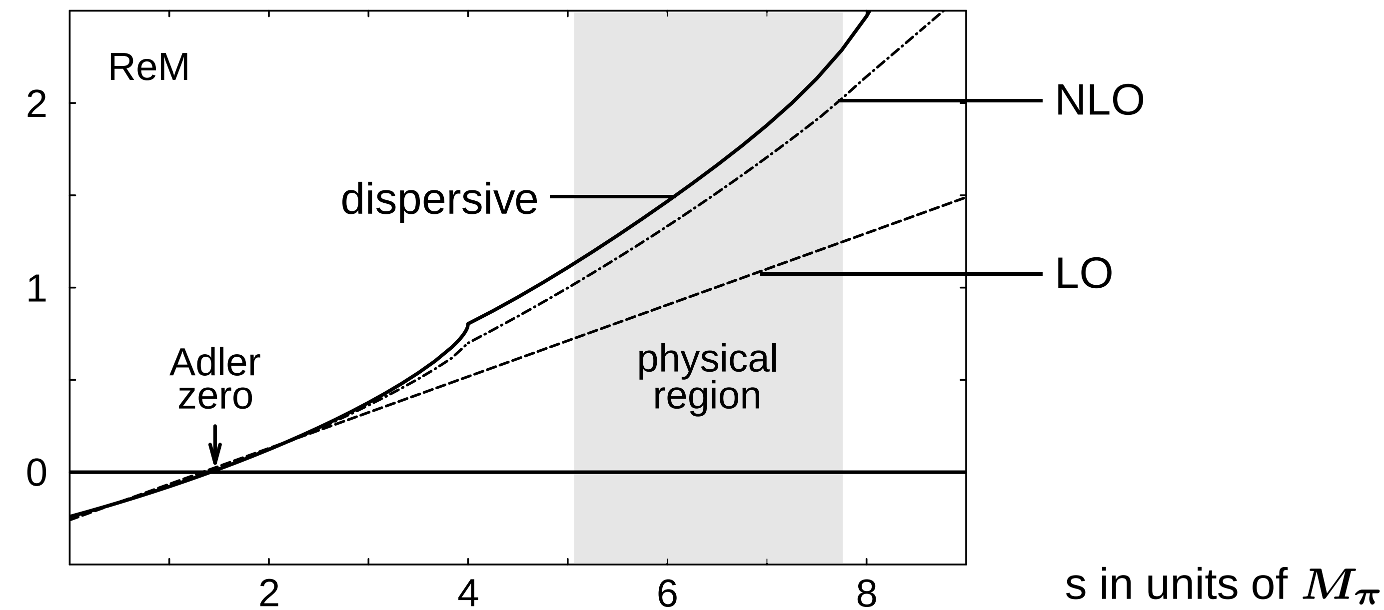


$\Rightarrow$ Final state interaction amplifies the transition.

- Effect of the higher order contributions on the result for Q is modest:

  $Q = 22.4 \pm 0.9$ Kambor, Wiesendanger & Wyler 1996

  $Q = 22.7 \pm 0.8$ Anisovich & L. 1996

  Confirms the one loop result, $Q = 22.9^{+2.1}_{-1.6}$, uncertainty reduced by a factor of 2.

- KWW also investigated the transition $\eta \to 3\pi^0$, predicted the slope of the corresponding Dalitz plot and showed that the result for the branching ratio $\Gamma_{\eta\to 3\pi^0}/\Gamma_{\eta\to\pi^+\pi^-\pi^0}$ is consistent with experiment.

Recent work on $\eta \to 3\pi$

- In the meantime, the experimental situation improved a lot: KLOE, MAMI, WASA.
- At low energies, the $\pi\pi$ phase shifts are now known to remarkable accuray:
  - Low energy precision experiments (E865, NA48, DIRAC).
  - Low energy theorems for scattering lengths.
  - Dispersion theory (Roy equations).
- Simulations of QCD on a lattice now reach sufficiently small quark masses. Powerful source of information, in particular also for the quark masses.
- For η decay, χPT has been worked out to NNLO. Bijnens & Ghorbani 2007
- At the precision reached, isospin breaking needs to be accounted for. Ditsche, Kubis & Meissner 2009
- Nonrelativistic effective theory. Gullström, Kupsc & Rusetsky 2009 Schneider, Kubis & Ditsche 2011
- Improved dispersive analysis, comparison with experiment
  - Diploma work of Manuel Walker 1998, PhD thesis of Stefan Lanz 2011. Thorough investigation in this framework is close to completion. Colangelo, Lanz, L. & Passemar
  - Entirely different approach: Kampf, Knecht, Novotny & Zdrahal 2011

Excursion: $\pi\pi$ scattering

- The interaction among the pions plays a central role at low energies, particularly when looking for physics beyond the Standard Model (precision).
- Dispersion theory of the $\pi\pi$ scattering amplitude: Roy equations. Roy 1971
- In the isospin limit, the Roy equations are exact.
  Inelastic processes such as $\pi\pi \to K\bar{K} \to \pi\pi$ are explicitly accounted for.
- Dispersion relations involve two subtractions, integrals converge rapidly.
  Subtraction constants can be identified with the S-wave scattering lengths, a_0, a_2.

$\Rightarrow$ If a_0, a_2 are known, the scattering amplitude can be calculated very accurately.

Ananthanarayan, Caprini, Colangelo, Gasser, L.
Descotes, Fuchs, Girlanda, Moussallam, Stern
García-Martín, Kamiński, Nebreda, Peláez, Ríos, Ruiz de Elvira, Ynduráin

Scattering lengths

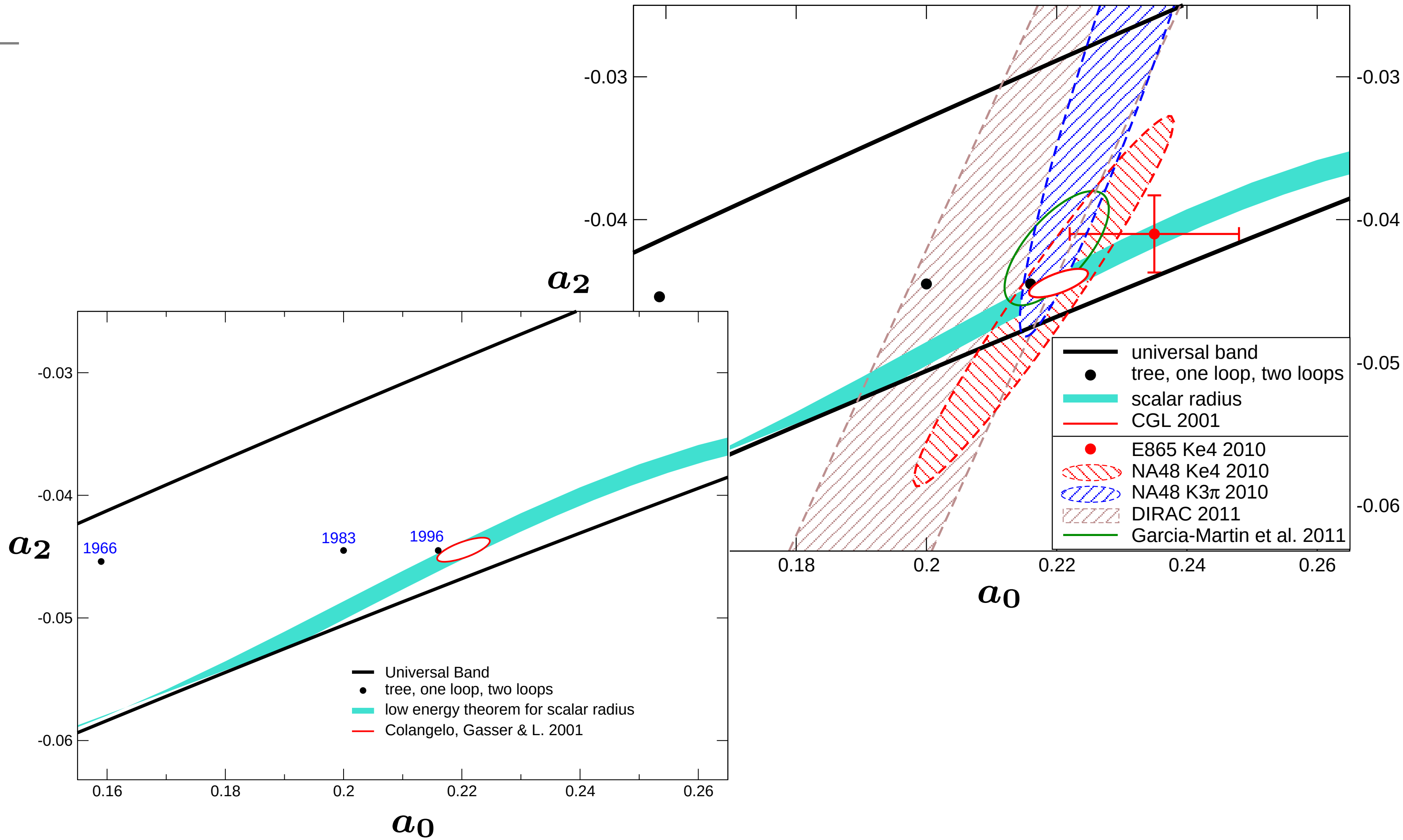
- Prediction at leading order of χPT:

$$a_0 = \frac{7M_\pi^2}{32\pi F_\pi^2} = 0.16, \qquad a_2 = -\frac{M_\pi^2}{16\pi F_\pi^2} = -0.045$$

Weinberg 1966

- χPT allows to analyze the contributions of higher order. Chiral expansion has been worked out to NNLO. Using dispersion theory, this leads to remarkably sharp predictions for a_0, a_2, which triggered new low energy precision experiments:
  - $\pi^+\pi^-$ atoms, DIRAC.
  - $K^\pm \to \pi^0\pi^0\pi^\pm$, $K^0 \to \pi^0\pi^0\pi^0$: cusp near threshold, NA48/2.
  - $K^\pm \to \pi^+\pi^-e^\pm\nu$ data: E865, NA48/2.

Experimental tests of the prediction



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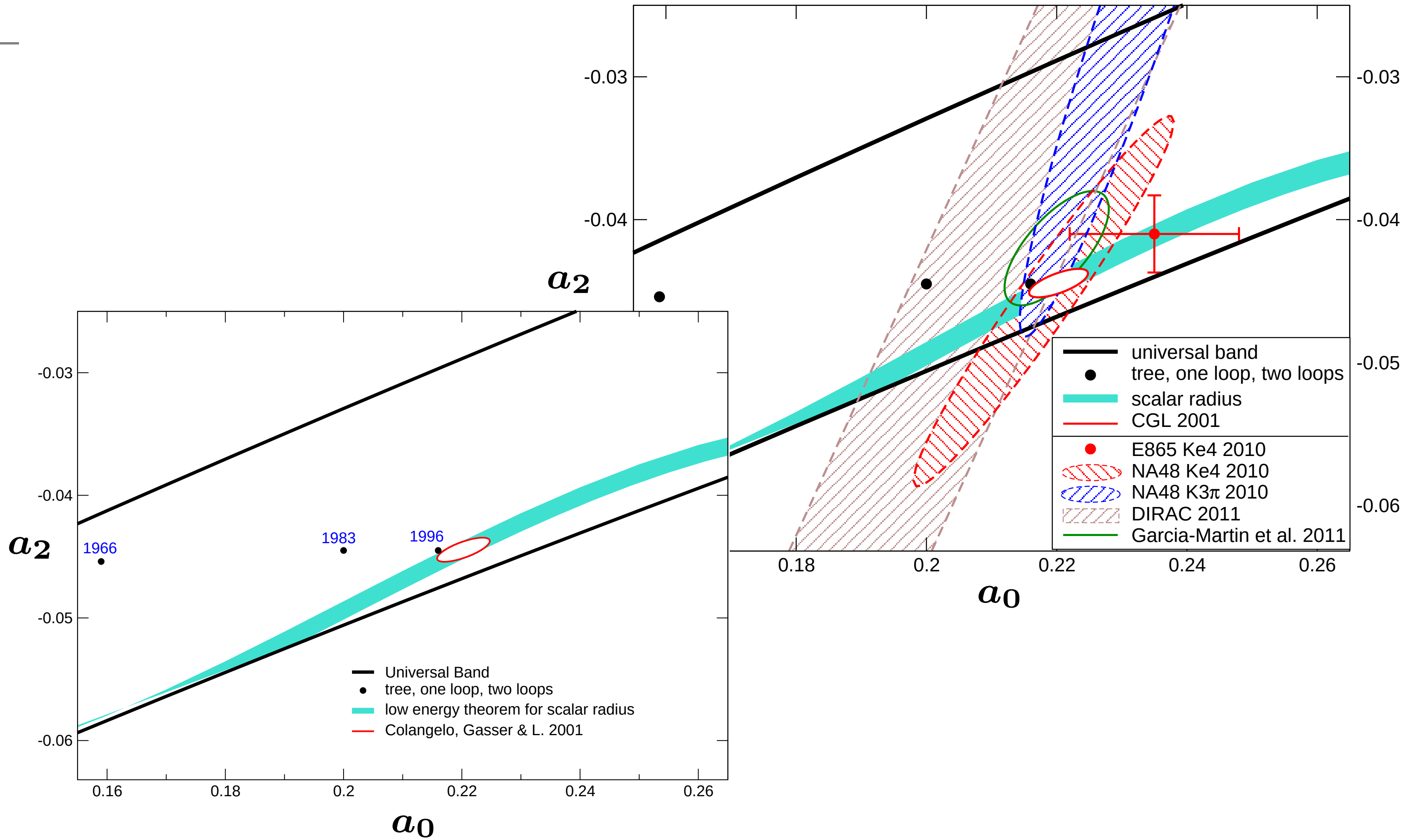
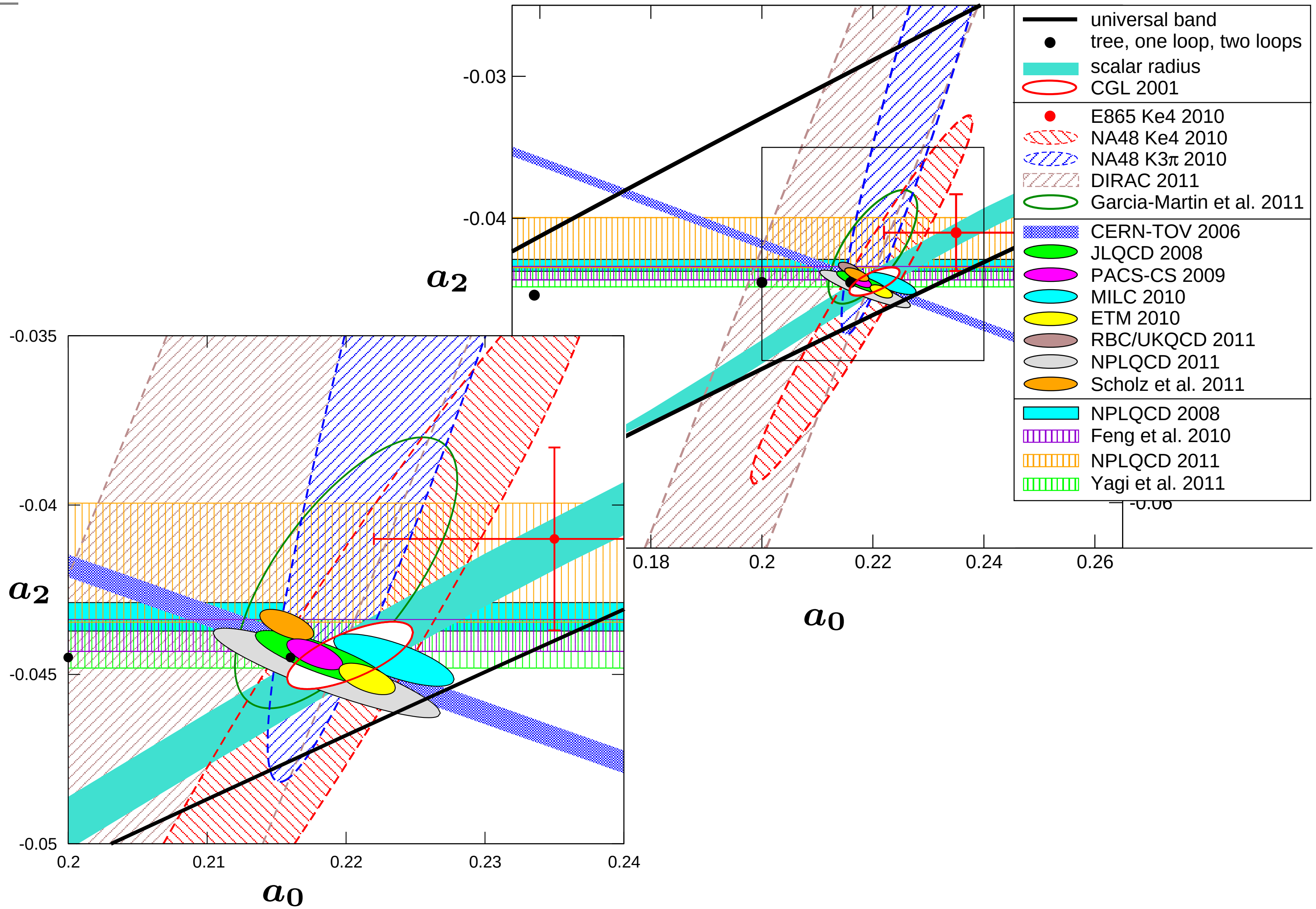


- Uncertainty in χPT prediction for a_0, a_2 is dominated by the uncertainty in the relevant coupling constants of the effective Lagrangian at NLO. These can now reliably be determined on the lattice, from the quark mass dependence of M_π and F_π.
- Direct determination of a_2 via dependence of the energy levels on the size of the box.

Compare the lattice results with prediction and experiment



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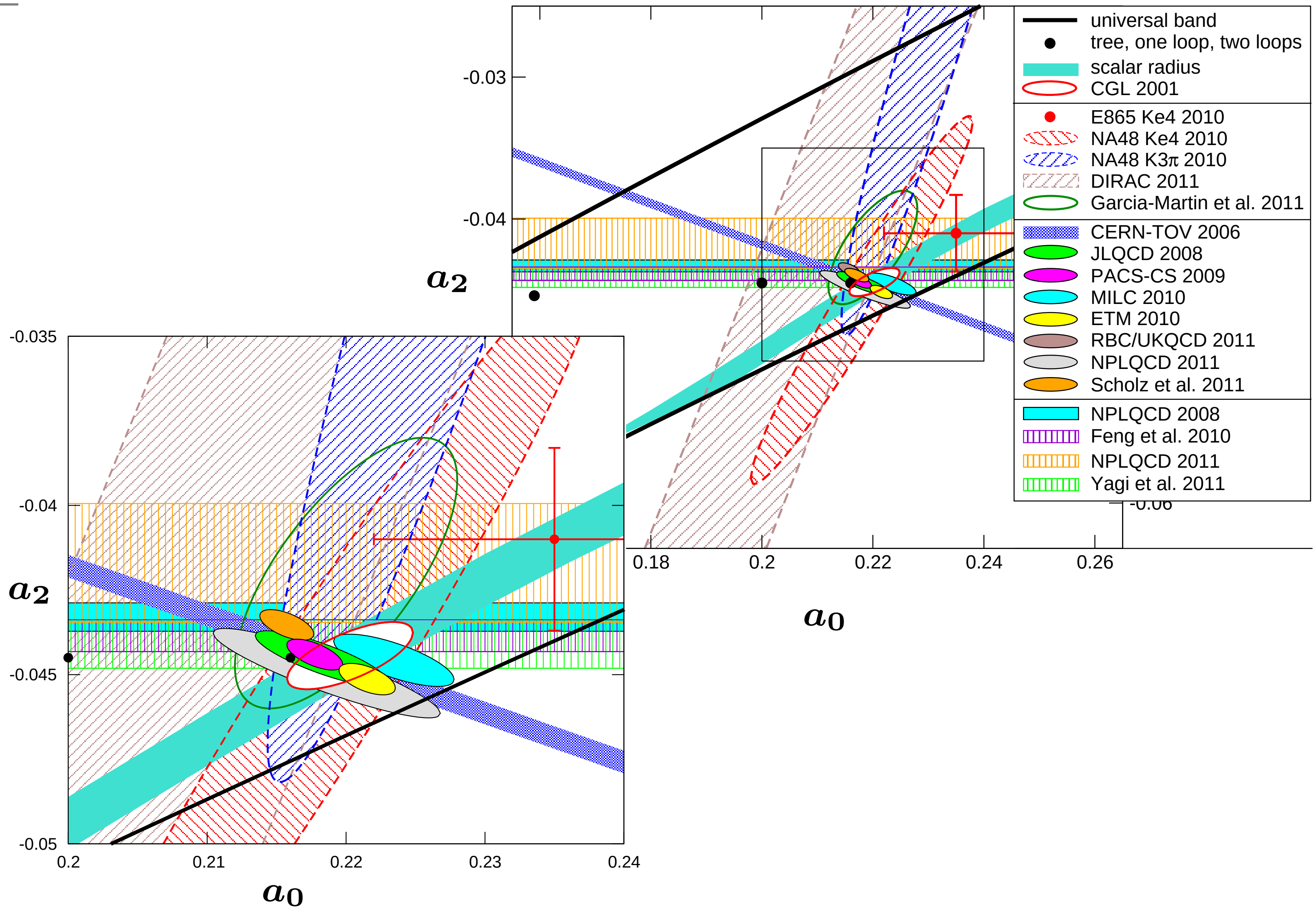

Compared to this, $\eta \to 3\pi$ is yet an underdeveloped country.

Back to $\eta \to 3\pi$

- Basic property of the dispersion relations: if the phase shifts are known, the amplitude is uniquely determined by the subtraction constants.
- If $M^{(1)}(s,t,u)$ and $M^{(2)}(s,t,u)$ are solutions, then $\lambda_1 M^{(1)} + \lambda_2 M^{(2)}$ is also a solution: the solutions form a linear space.

$\Rightarrow$ General solution is a linear superposition of basis functions. Number of independent basis functions is determined by the number of subtractions made.

- The subtraction constants are estimated with the following input:
  1. Measured Dalitz plot distributions of the charged and neutral decay modes.
     - Andrzej Kupsc (KLOE), Sergey Prakhov (MAMI) and Patrik Adlarson (WASA) kindly provided us with data tables.
     - In addition, we use the value for the slope of the Z-distribution of the neutral decay mode quoted by the PDG: $\alpha = -0.0317(16)$.
  2. The dispersive representation is matched with χPT at small values of s, t or u, where the higher orders of the chiral perturbation series are smallest.

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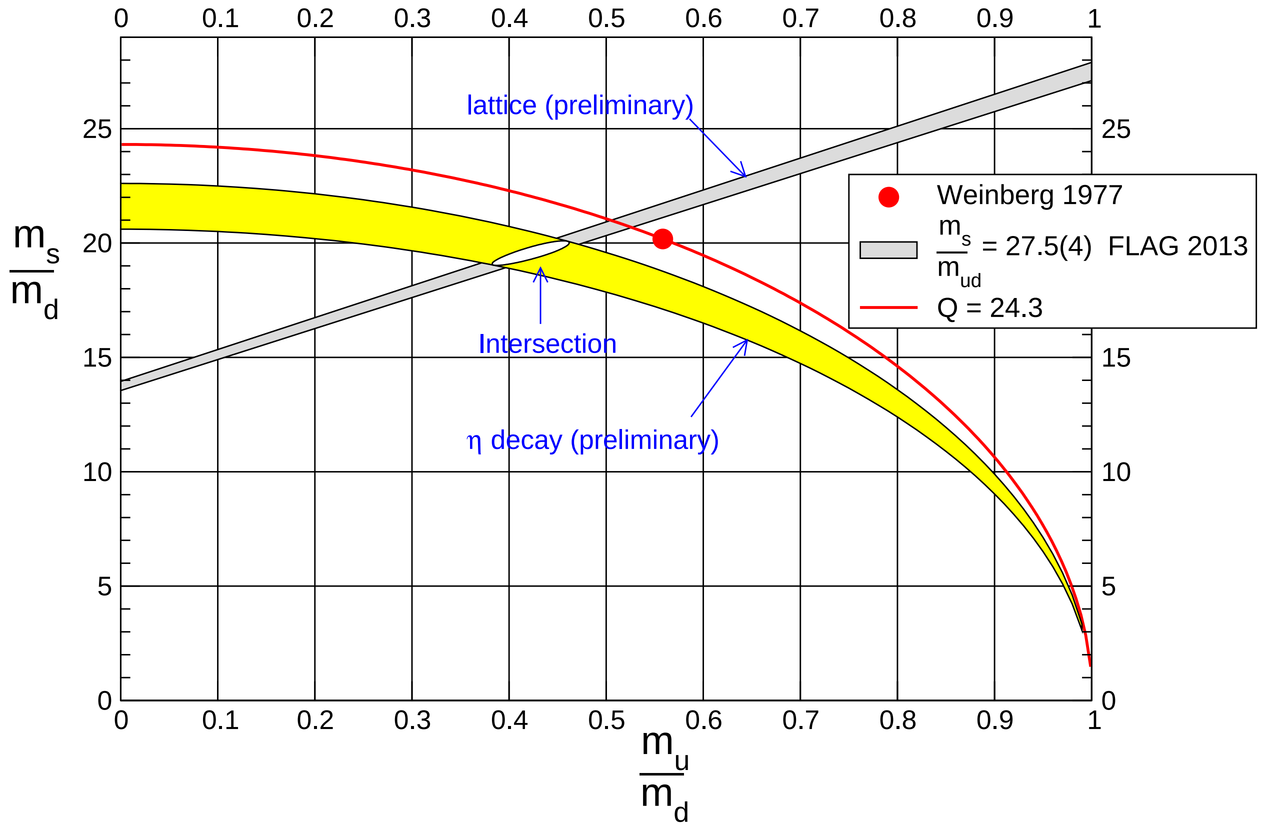
The devil sits in the details also in this case ... We are still not through with the error analysis.

To be done

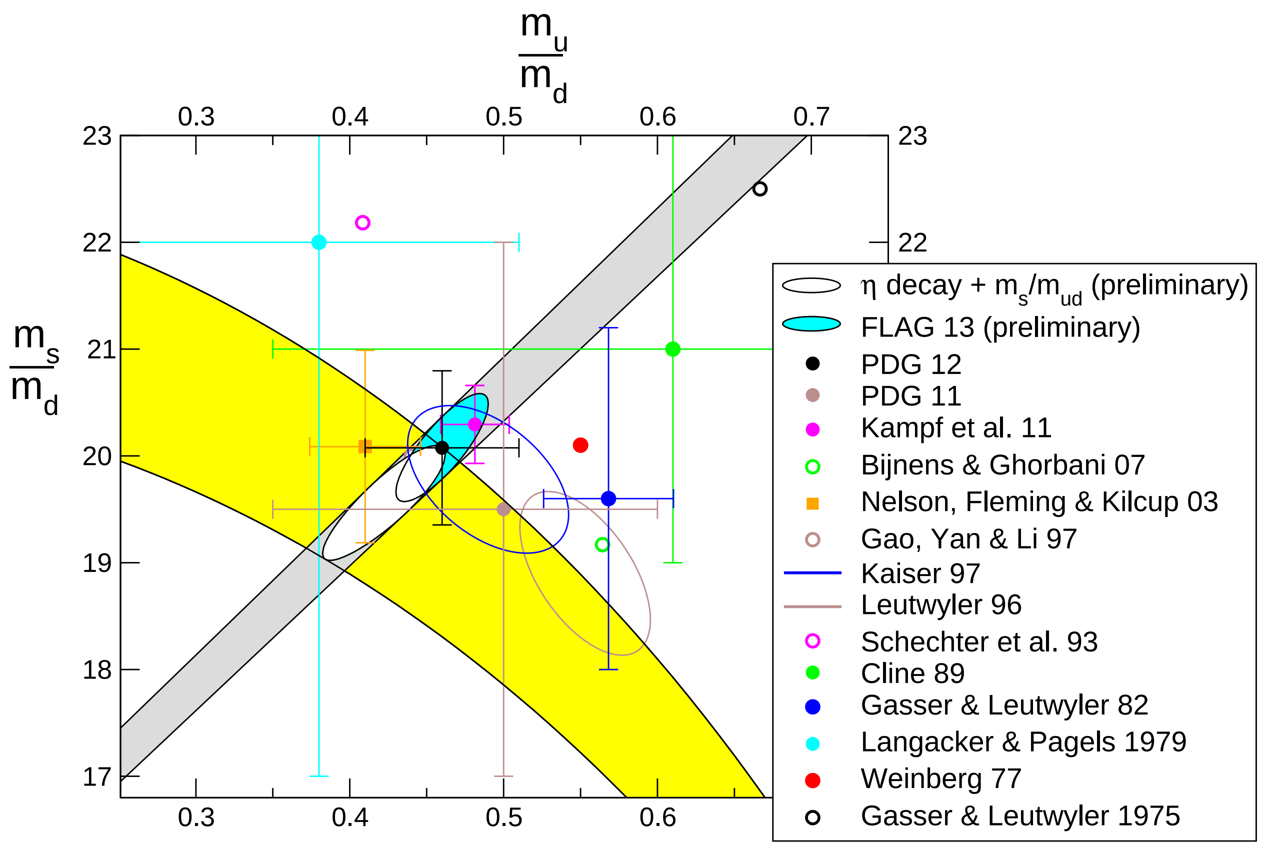
- The dependence of the result on the uncertainties in the input (phase shifts, subtraction constants, experimental errors) can be worked out explicitly, but we yet need to do this.
- Concerning the input used for the phase shifts, the Roy equations provide a reliable handle on the uncertainties – in their domain of validity: $\sqrt{s} \leq 1.15$ GeV.
- The dispersion integrals extend to ∞, but with the number of subtractions we are using, the contributions from the region above $K\bar{K}$ threshold are tiny.
- We approximate the isospin breaking effects by means of χPT, using the NLO representation of Ditsche, Kubis and Meissner.
- The value of Q can be determined either from the rate of the transition $\eta \to \pi^+\pi^-\pi^0$ or from $\eta \to 3\pi^0$. This offers a good test: evaluating the isospin breaking effects in this way, we indeed find that the two results for Q agree.
- I refrain from offering quantitative results for Q and for m_u/m_d and draw only qualitative conclusions. A detailed report on our work is forthcoming.

Consequence for the quark mass ratios



- Intersection moves to values of m_u/m_d and m_s/m_d that are somewhat smaller than those obtained with the LO mass formulae of Weinberg.

Comparison with other work



- The preliminary results for the ratio m_u/m_d are consistent with the lattice averages quoted by FLAG, but tend to be somewhat smaller.