The QCD critical point

Saumen Datta, Rajiv Gavai, Sourendu Gupta

ILGTI, TIFR Mumbai

6 August, 2013
XQCD 2013, Bern, Swaziland
1 Introduction

2 The susceptibilities

3 Critical behaviour

4 Summary
1 Introduction

2 The susceptibilities

3 Critical behaviour

4 Summary
EOS at $\mu \neq 0$

\[ \Delta P = P(\mu, T) - P(0, T). \]

Perform a series expansion of the pressure in powers of chemical potential

$$\Delta P(\mu_u, \mu_d, T) = \sum_{m,n} \chi_{m,n}(T) \frac{\mu_u^m \mu_d^n}{m!n!}.$$ 

Does this converge? Can one reconstruct the function? Well studied classical problem. Special complications: few coefficients known, with errors.

Simplest part of the problem: estimate whether the series is summable, radius of convergence and location of nearest singularity. Next more complicated: estimating value of the function, nature of divergence.
Perform a series expansion of the pressure in powers of chemical potential

\[
\Delta P(\mu_u, \mu_d, T) = \sum_{m,n} \chi_{m,n}(T) \frac{\mu_u^m \mu_d^n}{m! n!}.
\]

Does this converge? Can one reconstruct the function? Well studied classical problem. Special complications: few coefficients known, with errors.

Simplest part of the problem: estimate whether the series is summable, radius of convergence and location of nearest singularity. Next more complicated: estimating value of the function, nature of divergence.

Also, expansion in \( z = \frac{\mu_B}{T} \)

\[
\chi_B(\mu_B, T) = \frac{\partial^2 \Delta P}{\partial \mu_B^2} = \chi^0_B(T) + \frac{T^2}{2!} \chi^2_B(T) z^2 + \frac{T^4}{4!} \chi^4_B(T) z^4 + \cdots
\]
Our data

Lattice simulations with $N_f = 2$ staggered quarks and Wilson action. Used $N_t = 8, 6$ and 4; $m_\pi \simeq 0.3m_\rho$ MeV; spatial size $L = 4/T$.

Temperature scale, $T_c$, found by the point at which $\chi_L$ peaks. If $T_c \simeq 170$ MeV, then $1/a = 1.36$ GeV.

Configurations: 50K+ at each coupling; large number of fermion sources used for determination of fermion traces.

Errors depend on number of fermion sources for evaluation of propagator as well as number of gauge configurations. Multiple fermion loops are source hungry.
Numerical errors

Errors depend on number of fermion sources for evaluation of propagator as well as number of gauge configurations. Multiple fermion loops are source hungry.
Susceptibilities at $\mu = 0$

The QCD critical point
Susceptibilities at $\mu = 0$
Lattice spacing effects

The QCD critical point
Nearing continuum physics

Continuum: $T_c = 170$ MeV; p4 with $N_t = 8$: $T_c = 180$ MeV.

HTL, DR: Andersen et al, 1307.8098; NLO: Haque et al, 1302.3228; HotQCD: Petreczky, Lattice 2013
Continuum: $T_c = 170$ MeV; p4 with $N_t = 8$: $T_c = 180$ MeV.

HTL, DR: Andersen et al, 1307.8098; NLO: Haque et al, 1302.3228; HotQCD: Petreczky, Lattice 2013

Nearing continuum physics
The radius of convergence

For $N_t = 6$, $\mu_E / T_E = 1.7 \pm 0.1$ \cite{Gavai, SG: 2008}
1 Introduction
2 The susceptibilities
3 Critical behaviour
4 Summary
Must resum a series expansion

Truncated series sum is regular even at the radius of convergence, so it is missing something important.
Critical behaviour of $m_1$

If $\chi_B(z) \simeq (z_* - z)^{-\psi}$, then $m_1 = d \log \chi_B / dz$ has a pole. Series expansion of $\chi_B$ gives series for $m_1$. Resum series into a Padé approximant:

$$[0, 1] : m_1(z) = \frac{c}{z_* - z}$$

Width of the critical region? If we define it by

$$\left| \frac{m_1(z)}{m_1(0)} \right| > \Lambda,$$

then $|z - z_*| \leq z_*/\Lambda$.

Errors in extrapolation? We have

$$\left| \frac{\Delta m_1}{m_1} \right| > \frac{1}{1 - \Lambda \delta},$$

where $\delta$ is fractional error in $z_*$. 

ILGTI The QCD critical point
Critical slowing down

The QCD critical point
At a critical point

\[ \chi_B = \frac{\partial^2 (P/T^4)}{\partial z^2} \approx (z^2 - z^2)^{-\psi}. \]

Continuity and finiteness of \( P \) at the CEP forces \( \psi \leq 1 \).

Since

\[ m_1(z) = \frac{d \log \chi_B}{dz} \approx \frac{2 \psi z}{z^2_* - z^2}, \]

use the series to estimate the critical exponent. Series for \( m_1 \) has one term less than series for \( \chi_B \).

Accurate results require fine statistical control of at least 3 series coefficients of \( \chi_B \): 2 of \( m_1 \).
Widom scaling

Widom scaling for the order parameter gives

\[ |\Delta \mu| = |\Delta n|^\delta J \left( \frac{|\Delta T|}{|\Delta n|^{1/\beta}} \right), \]

where \( \Delta T = T - T_E \) and \( \Delta \mu = \mu - \mu_E \). For \( \Delta T = 0 \) one finds \( \Delta n \propto |\Delta \mu|^{1/\delta} \) in the high density phase. Then clearly one has

\[ \psi = 1 - \frac{1}{\delta}. \]

For the 3d Ising model, \( \delta = 1.49 \), so \( \psi = 0.79 \). Since the identification of the two scaling directions is arbitrary, one can vary these. This gives \( 0.79 \leq \psi \leq 1 \).

In mean field theory one has \( \delta = 3 \), so \( 0.66 \leq \psi \leq 1 \). The data cannot yet distinguish between these cases.
Testing the DLOG Pade

Test resummation by using 3rd term of $m_1$. 

Unconstrained DLOG Pade
Ising-constrained DLOG Pade
Large errors in $\psi$, but $\psi < 1$ as expected from continuity of pressure. Ising prediction: $\psi \geq 0.79$. 
The pressure

The QCD critical point
Introduction

The susceptibilities

Critical behaviour

Summary
Critical point and critical region

Critical point estimates:
- Budapest-Wuppertal Nt=4
- ILGTI Nt=8
- ILGTI Nt=6
- ILGTI Nt=4

Freezeout curve

T/Tc vs. μ_B/T

10 GeV
30 GeV

The QCD critical point
Critical point and critical region

The QCD critical point

Freezeout curve

$30 \text{ GeV}$

$10 \text{ GeV}$
Critical point and critical region

Freezeout curve

T/Tc vs. \( \mu_B/T \)
Critical point and critical region

The QCD critical point

Freezeout curve

T/T_c vs. \mu_B/T

30 GeV

10 GeV

ILGTI

The QCD critical point