Calculating $\hat{q}$ using EQCD simulations

Marco Panero (Helsinki), Kari Rummukainen (Helsinki) and Andreas Schäfer (Regensburg)

UNIVERSITY OF HELSINKI

HELSINKI INSTITUTE OF PHYSICS

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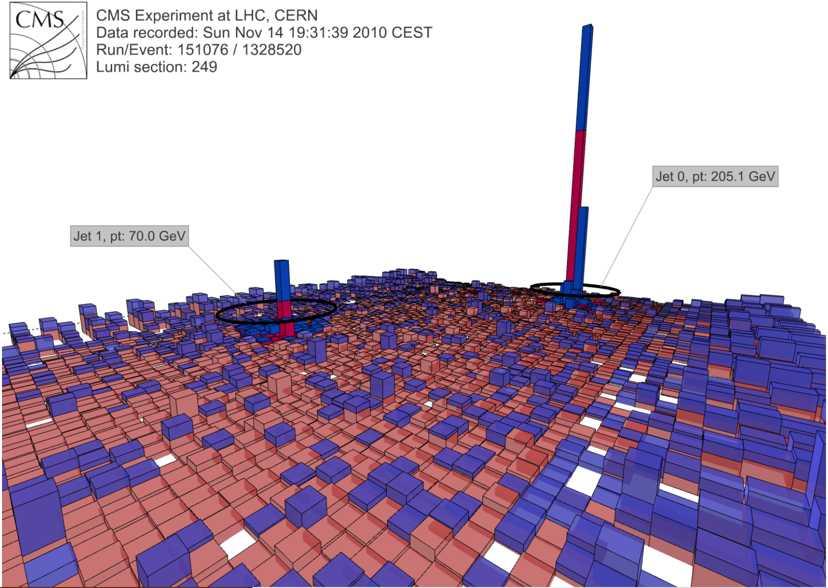
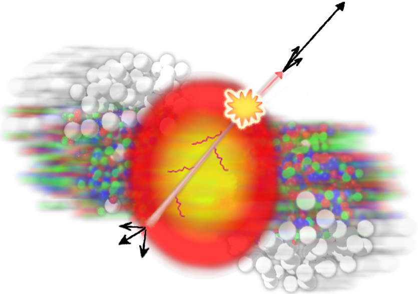
Jet quenching: back-to-back jets in A-A collisions



Jet quenching

Jet quenching is evidence for strongly coupled quark-gluon plasma (QGP)



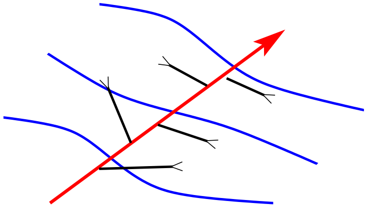
A fast parton

- is generated in a hard collision (large Q^2)
- Interacts with the expanding QGP
- hadronization into a jet

Parton-plasma cross-section $\sigma(q_\perp, Q^2)$
Here: study σ on the lattice using electrostatic QCD, EQCD

Hard parton propagation in QGP

- Multiple soft-scattering, eikonal approximation ($v = 1$)



- *Transverse* momentum broadening described by jet quenching parameter: [Baier et al.]

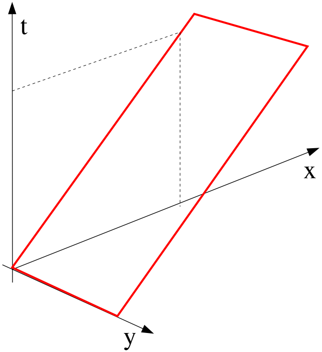
$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L}$$

- Can be evaluated in terms of a *collision kernel* $C(p_\perp)$

$$\hat{q} = \int^\Lambda \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)$$

Light-like Wilson loop

- The collision kernel is related to the lightlike Wilson loop:



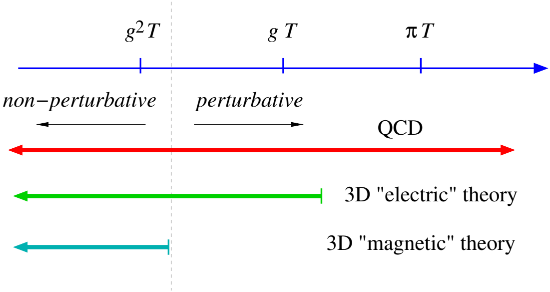
- The collision kernel $C(p_\perp)$ is known to leading order [Arnold,Xiao] and next-to-leading order [Caron-Huot].
- At higher orders non-perturbative effects contribute $\rightarrow$ lattice simulations?

Scale hierarchies and effective theories

- At high T, QCD has 3 distinct scales:

$$g^2 T/\pi \text{ (ultrasoft)} \ll gT \text{ (soft)} \ll \pi T \text{ (hard)}$$

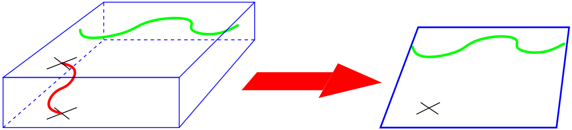
- Hierarchy of effective theories (for static quantities) by successive "integration" over hard modes:
  - Scales $p \lesssim gT$: Electrostatic QCD, EQCD
  - scales $p \lesssim g^2 T$: Magnetostatic QCD, MQCD



EQCD

- Starting from Euclidean (continuum) QCD with N_f quarks, integrate modes $p \gtrsim T$: *fermions, non-zero Matsubara frequencies*

$\rightarrow$ 3d effective theory (dimensional reduction)

1/ T 

EQCD action:

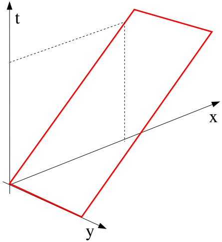
$$\mathcal{L}_{\rm EQCD} = \frac{1}{4} F^a_{ij} F^a_{ij} + {\rm Tr}\left((D_i A_0)^2\right) + m_{\rm E}^2 \,{\rm Tr}\left(A_0^2\right) + \lambda_3 \left({\rm Tr}\left(A_0^2\right)\right)^2$$

[Braaten and Nieto; Kajantie, Laine, K.R., Shaposhnikov]

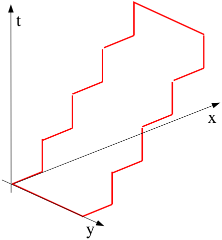
- Parameters $g_{\rm E}^2$, $m_{\rm E}^2$, λ_3 depend on g^2, N_f and T.
- Used succesfully in calculations of pressure, screening lengths and susceptibilities in hot QCD
- Superrenormalizable: lattice counterterms known

Return to $\hat{q}$: how to compute $C(p_\perp)$ on the lattice?

- Minkowski space (real-time) object $\rightarrow$ Minkowski lattice? Not possible!



$\Rightarrow$

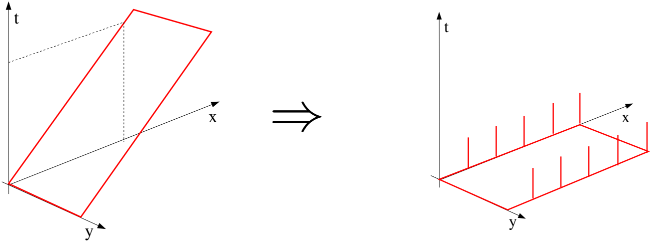


- Use std. Euclidean finite-T lattice? [Majumder] Tricky (and costly) analytical continuation required [Laine; Laine and Rothkopf]
- Classical field theory simulation? [Laine and Rothkopf]
  - Minkowski
  - Captures (static) g^2T physics correctly
  - Treats hard modes incorrectly
- Calculation using EQCD?

Evaluating $C(p_\perp)$ with EQCD

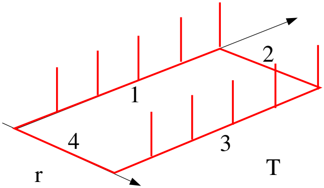
- Intuitively: **soft physics is slow physics**
- Overdamped evolution
- Soft fields along the light cone $\sim$ soft fields along $t = \mathrm{const}$ plane

$\Rightarrow$ Can evaluate $C(p_\perp)$ using static EQCD [Caron-Huot; Aurenche,Gelis,Zaraket]



- The decorations on x-direction lines are insertions of temporal "parallel transporters", constucted from Euclidean→Minkowski rotated A_0's.
- Shown rigorously by [Caron-Huot; Ghiglieri et al.]

Evaluating $\hat{q}$ with EQCD



More precisely: construct "potential" $V(r)$ from generalised Wilson loop

$$
\begin{aligned}
\exp(-V(r)T) &= \mathcal{W}(r,T) = \mathrm{Tr}\, L_1 L_2 L_3^\dagger L_4^\dagger \\
L_1 &= U_x(0,0)\, H(a,0)\, U_x(a,0)\, H(2a,0) \ldots U_x(T-a,0)\, H(T,0) \\
L_2 &= U_y(T,0) U_y(T,a) \ldots U_y(T,r) \\
L_3 &= U_x(0,r)\, H(a,r) \ldots U_x(T-a,r)\, H(T,r) \\
L_4 &= U_y(0,0) \ldots U_y(0,r)
\end{aligned}
$$

where $U_x \in \mathrm{SU}(3)$ is the standard lattice x-direction link matrix and

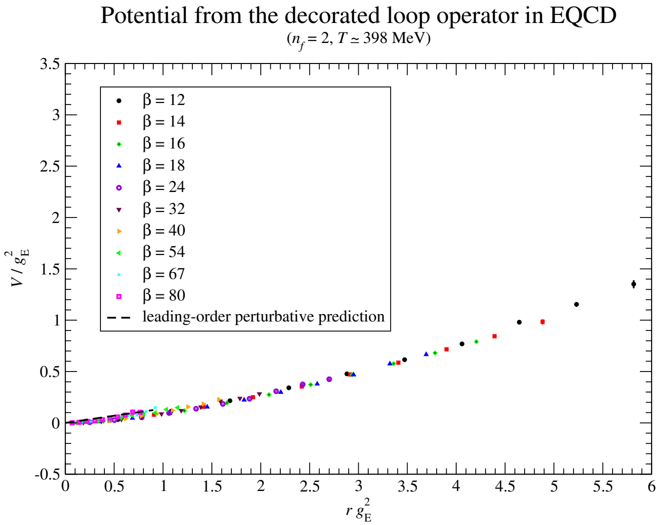
$$H(x) = \exp(a g_{\mathrm{E}} A_0)$$

is a *Hermitean* Wick-rotated (Euclidean→Minkowski) "parallel transporter"

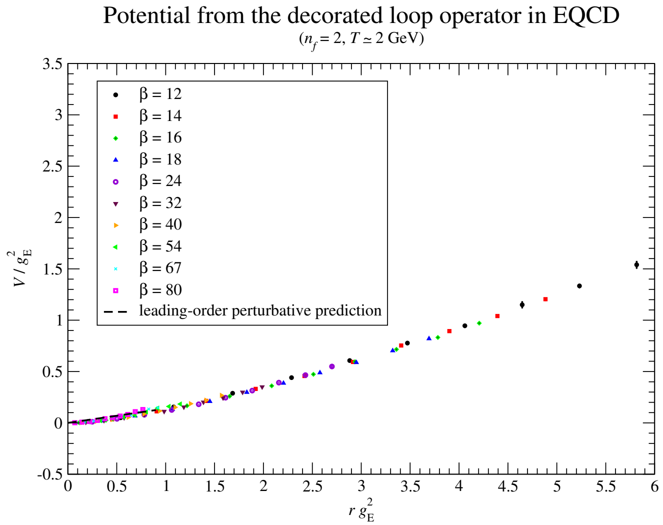
Measurements

- Lattice spacings used: $ag_{\mathrm{E}}^2 = 0.5 \ldots 0.075$ ($beta = 12 \ldots 80$)
- Volumes up to $120^2 \times 168$
- Loops are measured using a modified version of the multi-level algorithm [Lüscher and Weisz] $\rightarrow$ large loops possible, accurate results.
- Two temperatures: $T = 398$ MeV and 2 GeV
- We also measure std. Wilson loop in MQCD (3D pure gauge theory)

$V(r)$ at $T \approx 398$ MeV



$V(r)$ at $T \approx 2\,\text{GeV}$



Extracting $\hat{q}$ from $V(r)$

- No sign of the "Coulomb" term in the potential $V(r_\perp)$
- $C(p_\perp)$ is 2d Fourier transform of $-V(r_\perp)$
- $\hat{q}$ can now be in principle obtained from

$$\hat{q} = \int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp) = \int d^2 r_\perp \nabla^2 V(r_\perp)$$

  (+ suitable cut-offs needed)
- Good fits to $V(r_\perp)$ are obtained with the perturbatively motivated ansatz

$$V(r_\perp)/g_{\mathrm{E}}^2 = Ar_\perp + Br_\perp^2 + Cr_\perp^2 \ln(g_{\mathrm{E}}^2 r)$$

  in the range $0.3 \leq g_{\mathrm{E}}^2 r_\perp \leq 3$, with A fixed to perturbative estimate.
- The ansatz enables us to integrate $\hat{q}$
- We subtract the perturbative LO and NLO contributions as was done by Laine in MQCD (valid when $p_\perp < m_{\mathrm{E}}$)

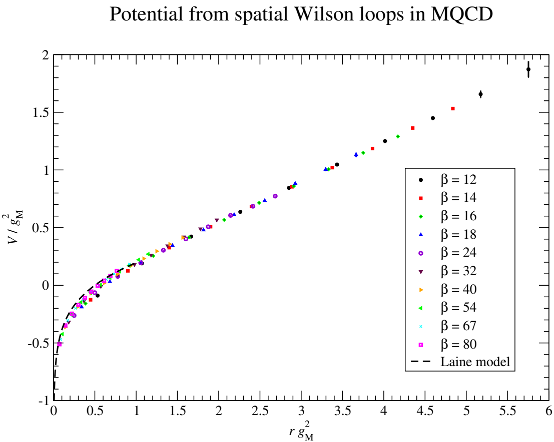
Results

- The non-perturbative EQCD contribution is

$$\delta\hat{q}_{\rm EQCD} \simeq \begin{cases} 0.55(5)g_{\rm E}^6 & \text{for } T \simeq 398 \text{ MeV} \\ 0.45(5)g_{\rm E}^6 & \text{for } T \simeq 2 \text{ GeV} \end{cases}$$

- Comparable to perturbative NLO result $\sim 0.47 g_{\rm E}^6$ (which, in turn, is large compared to LO term)
- *Approximate* estimate: $\hat{q} \sim 6$ GeV^2/fm at RHIC temperatures

$V(r)$ from MQCD



- Missing electric sector A_0 and electric-magnetic interactions
- The result by [Laine] fits data well: $\delta\hat{q} \sim 0.08 g_{\rm E}^6$

Conclusions

- First tentative results for $\hat{q}$ from EQCD
- In the same ballpark than results from
  - perturbation theory
  - holography [Liu, Rajagopal, Wiedemann]
  - experimental analysis [Eskola et al.]
- Full analysis, including continuum limits and sensitivity tests to various cutoffs still to be done.
- Improved Wilson loop operator? [D'Onofrio et al.]