Photons and Transport at NLO

with Jacopo Ghiglieri, Juhee Hong, Aleksi Kurkela, Egang Lu, Derek Teaney

• Photons: motivation and basics
• Convergence of Perturbation Theory
• When soft physics is light-cone physics
• When light-cone physics is thermodynamics
• NLO photon production: results, prospects
Stages of a Heavy Ion Collision

1. Ions collide, making $q, g, \text{photons}$ “primary”
2. $q, g$ rescatter as QGP, make $\text{photons}$ “thermal”?
3. Hadrons form, scatter, make $\text{photons}$ “Hadronic”
4. Hadrons escape, some decay to $\text{photons}$ “decay”

Photon re-interaction rare ($\alpha_{EM} \ll 1$): direct info.
Thermal photons $\textit{may}$ act as a thermometer for QGP.
Production rate is interesting! Mostly for $E > 2$ GeV, several $T$. 

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How Photons Get Made

Since $\alpha_{\text{EM}} \ll 1$, work to lowest order in $\alpha_{\text{EM}}$:

- assume photon production Poissonian

- neglect back-reaction on system cooling insignificant...

Single-photon production at $\mathcal{O}(\alpha_{\text{EM}})$

$$2k^0 \frac{d\text{Prob}}{d^3 k} = \sum_X \text{Tr} \rho U^\dagger(t) |X, \gamma(k)\rangle \langle X, \gamma(k)| U(t)$$

$U(t)$ time evolution operator, $\rho$ density matrix.
Expand $U(t)$ in EM interaction picture:

$$U(t) = 1 - i \int^t dt' \int d^3 x \ eA^\mu(x, t') J_\mu(x, t') + O(e^2)$$

$A^\mu$ produces the photon. Get

$$\frac{d\text{Prob}}{d^3 k} = \frac{e^2}{2k^0} \int d^4 Y d^4 Z e^{-iK \cdot (Y - Z)} \sum_X \text{Tr} \rho J^\mu(Y)|X\rangle\langle X|J_\mu(Z)$$

And $\sum_X |X\rangle\langle X| = 1$. Assume slow-varying, near-equilibrium: $\int d^4 Z \to Vt$: Get rate per 4-volume:

$$\frac{d\Gamma}{d^3 k} = \frac{e^2}{2k^0} G^<(K), \ G^<(K) \equiv \int d^4 Y e^{-iK \cdot Y } \langle J^\mu(Y) J_\mu(0)\rangle_\rho$$

Success of Hydro – but not true at early times ....
Calculational Approaches

No first-principles, nonperturbative tool for $\langle J^\mu J_\mu \rangle(K)$. Only

- Lattice techniques (uncontrolled analytic continuation. Avoid)
- Weak-coupling techniques (uncontrolled extrapolation from $\alpha_s < 0.1$. Pursue)
- Strong-coupling $\mathcal{N}=4$ SYM (uncontrolled relation to QCD. Avoid)

How bad is weak coupling?

- It fails at $T \sim$ few $T_C$ ?
- It fails at $T \sim 10^6 T_C$ ?
- It fails at all temperatures? Truth: some of each!
Lessons from the Pressure

Divide degrees of freedom in 2 groups: **Hard and Soft**

Naive order-by-order $g$-expansion

- Converges if $T > 10^6 T_C$
  - Arnold-Zhai, Braaten-Nieto, etc

Integrate out **Hard**, solve **Soft** nonperturbatively: 3-D theory!

- Works down to $T = 2 T_C$.
  - Kajantie et al, etc

Hard physics is perturbative. There is hope!

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Perturbative treatment

We want \( G^<(K) \equiv \int d^4 Y e^{-iK \cdot Y} \langle J^\mu(Y) J_\mu(0) \rangle_\rho \)

\( J^\mu = \bar{\psi} \gamma^\mu \psi \). Correlator of two quarks. Something like
Perturbative treatment

We want

\[ G^K(\mathcal{K}) \equiv \int d^4 Y e^{-i \mathcal{K} \cdot \mathcal{Y}} \langle J^\mu(Y) J_\mu(0) \rangle_\rho \]

\[ J^\mu = \bar{\psi} \gamma^\mu \psi. \] Correlator of two quarks. In general,

Worse: dynamics complex, no nice effective 3D theory!

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Start with Kinematics

\[
\gamma \text{ produc: } \sum_{\psi_f} \langle \psi_i | A^\mu \bar{\psi} \gamma_\mu \psi | \psi_f \rangle \langle \psi_f | A^\nu \bar{\psi} \gamma_\nu \psi | \psi_i \rangle
\]

In \( M \), \( \psi, \bar{\psi} \) momenta \( p, k - p \) must add to \( k \) of photon:

- **Black**: way off-shell, but big phase space
- **Blue**: less phase sp, but soft enhancement
- **Red**: both can be almost on-shell.

Call these regions Hard, **Soft**, and **Collinear**.
The $P_\perp$, $P_+$ plane:
Hard case

If all momentum components (transverse and longitudinal) are large, physics is simple: short distance-and-time correlators, PQCD works. Loop corrections are $\mathcal{O}(g^2)$ and should get large around $T \sim 2T_C$.

The challenge is the other two regions, where Pert. Thy. need not work as well.
Momentum-space vs Coordinate space

Momentum $K$ lightlike $\rightarrow$ lightlike $X$-separation:

$$f(K) = \int d^4X e^{-iK \cdot X} f(X)$$ involves all $X$

But $q, \bar{q}$ start and end at same place:

$q$ propagator

\begin{center}
\begin{tikzpicture}
\node (a) at (0,0) {$J^\mu(0)$};
\node (b) at (4,0) {$J^\mu(X)$};
\draw[thick,->] (a) .. controls (2,1) and (2,-1) .. (b);
\end{tikzpicture}
\end{center}

$q$ propagator

$X$ determined by Fourier properties of $P$ and $K - P$.

$P$ small (or $p_\perp$ small): $X$ (or $x_\perp$) large.
$K$ big, but $X$ in $\int d^4X \exp(-iK \cdot X)$ also big. How?

Need phase
$\exp(-iK \cdot X)$ small. Occurs in narrow region.
Write $t, z$ as
$X^- = (t - z)$,
$X^+ = (t + z)/2$.

Since $-K \cdot X = K^+X^- + K^-X^+$, $K^+$ big,
contribution is from region $X^- \simeq 0$ (Light Cone)
Lightcone correlators are Simple!

\[ x^- = 0 \ (x = t) \] is “Lightcone” of photon

Separation lightlike if \( x_\perp = 0 \), spacelike if \( x_\perp \neq 0 \).

Causality \( \rightarrow \) only pre-existing correlators.

Unequal times usually means Complicated Dynamics.

Now Complicated dynamics Simple Thermodynamics!

- Energy-dependent: Just Thermal Masses!
- Energy-independent: Classical (3-D theory) correlators!
Collinear case

Collinear ⇒ almost on-shell ⇒ large $x$ separation

$x^- \ll x_\perp \ll x^+ \ (1/T \ll 1/gT \ll 1/g^2 T)$

Consider *spacetime trajectory* of $q, \bar{q}$:

Need $x_\perp$-separated Wilson loop.

Spacetime picture pioneered by B. Zakharov, hep-ph/9607440,9807540
Nontrivial analysis B. Zakharov, BDMPS, AMY

\[
\frac{dN_{\gamma}}{d^3k d^4x} = \frac{\alpha_{\text{EM}}}{\pi^2 k} \int_{-k/2}^{\infty} \frac{dp^+}{2\pi} n_f(k+p) \left[ 1 - n_f(p) \right] \frac{p^2 + (p+k)^2}{2[p(p+k)]^2} \times \lim_{x_\perp \to 0} 2 \text{Re} \partial_{x_\perp} f(x_\perp)
\]

\[
2 \nabla_\perp \delta^2(x_\perp) = \left[ C(x_\perp) + \frac{ik}{2p^+(k+p^+)} (m_\infty^2 + \nabla_\perp^2 x_\perp) \right] f(x_\perp)
\]

\[f(x_\perp): \text{density matrix } |\psi_{P+K}\rangle \langle \gamma_K |\psi_P| \text{ or } |\psi_P \bar{\psi}_{K-P}\rangle \langle \gamma_K |\]

Eikonal evolution (Evolution in \(x^+\)) — \(x_\perp\) diffusion, AND

Wilson-loop interaction with medium \(C(x_\perp)\).
\( \mathcal{C}(x_{\perp}) \) is Euclidean!

\( \mathcal{C}(x_{\perp}) \): Wilson loop with space-separated lightlike lines. All points at spacelike or lightlike separation.

Soft contribution is **Euclidean**!!  

Calculate it with *simple* perturbation theory (EQCD)

Calculate it on the lattice?!

NLO corrections to \( \mathcal{C}(x_{\perp}) \) computed. NNLO would be nonperturbative; possible via lattice

S. Caron-Huot, 0811.1603

Rummukainen, next talk
Consider correlator $G^<(x^0, x)$ with $x^z > |x^0|$. Fourier representation

$$G^<(x^0, x) = \int d\omega \int dp_z d^2p_\perp e^{i(x^z p^z + x_\perp \cdot p_\perp - \omega x^0)} G^<(\omega, p_z, p_\perp)$$

Use $G^<(\omega, p) = n_b(\omega)(G_R(\omega, p) - G_A(\omega, p))$ and define $\tilde{p}^z = p^z - (t/x^z)\omega$:

$$G^< = \int d\omega \int d\tilde{p}^z d^2p_\perp e^{i(x^z \tilde{p}^z + x_\perp \cdot p_\perp)} n_b(\omega) \left( G_R(\omega, \tilde{p}^z + \omega x^0/x^z, p_\perp) - G_A \right)$$

Perform $\omega$ integral: upper half-plane for $G_R$, lower for $G_A$, pick up poles from $n_b$:

$$G^<(x^0, x) = T \sum_{\omega_n = 2\pi nT} \int dp^z d^2p_\perp e^{i\mathbf{P} \cdot \mathbf{x}} G_E(\omega_n, p_z + i\omega_n(x^0/x^z), p_\perp)$$

Large separations: $n \neq 0$ exponentially small. $n = 0$ contrib. is $x^0$ independent!
Start with brute force: do the diagrams

Cut hard line: $p^- \simeq 0$, hard-line approx. $p^+$ independent.

Remaining integrals (using KMS) $(P, Q$ are resp. soft quark, gluon momenta)

$$
\int_{\sim g_T} d^2 p_\perp dp^+ \int_{\sim g_T} d^4 Q n_b(k^0) (G_R - G_A)
$$

$G_R$: retarded function of sum of all 4 diagrams’ guts.

Momentum $p^+$ is **null**. Any $R/A$ function is analytic in upper/lower half plane for time-like or **null** $p$-variable.

Analytically continue in $p^+!!$

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Deform $p^+$ contour into complex plane

Now $p^+ \gg p_\perp, Q$. (On mass-shell) Expand in $p^+ \gg p_\perp, Q$

$$G_R[4 \text{ diagrams}] = C_0(p^+)^0 + C_1(p^+)^{-1} + \ldots$$

$C_0$ is on-shell width, gives linear in $p^+$ divergence.
$C_1$ is on-shell dispersion correction, $dp^+ / p^+$ gives const.
Huh? Continuation possible because $J^\mu$ light-cone separated. And light-cone correlators are simple!

- $C_0$ term: Exactly the limit of collinear calculation when one quark momentum gets small. Already included.

- $C_1$ term: real dispersion-correction. Really simple:

$$\gamma\text{-rate} \propto \int \frac{d^2 p_\perp}{(2\pi)^2} \frac{m^2_\infty}{p_\perp^2 + m^2_\infty}$$

where $m^2_\infty$ is dispersion correction. Has leading-order piece (hard modes) and subleading piece (dispersion correction of soft modes). both are known.
Remaining region—similar story. Null-separation physics, all condensates.

Summing it up: two corrections

Upward correction: more scattering at NLO.
Downward correction: fewer soft gluons, less dispersion corr.
Numerical conspiracy: effects nearly cancel [Accidental!!]

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Main lesson

All the sticky IR physics shows up in a few condensates. Some are dispersion corrections – physically simple. Some are Euclidean – get directly on the lattice.

Bad news: $O(g)$ corrections big even for $\alpha_s = 0.1$ or $1000\ T_c$. Sort of expected that.

Good news: A few condensates. Determine them nonperturbatively, maybe get down to few $T_c$?

Get them on the lattice?
\( \mathcal{C}(x_\perp) \) on the lattice

Short side: \( x_\perp \) Wilson line \( \exp \int iA_\perp \cdot x_\perp \Rightarrow U_\perp U_\perp \ldots \)

Long side: \( x^+ \) Wilson line \( \exp \int i(A^z + A^0)dz \Rightarrow U_z e^{a\Phi} U_z e^{a\Phi} U_z \ldots \)

The latter is a new beast. Lattice renormalization properties?

Under investigation.

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Other transport coefficients?

We want Baryon Diffusion $D$ and (especially) shear $\eta$!
Both controlled by high-energy $E = \text{several} \; T$ particles
Lightlike correlators should again dominate:

\[ T^{xy} \text{ disturbs a particle} \]

It induces another $T^{xy}$

NLO effects arise along particle’s lightlike trajectory.
Problem: transfer of stress to someone else

Lines are timelike separated

There are $O(g)$ corrections
Conclusions

• Photon production is worth computing
• “Enhanced” Pert. calculation – few $T_C$??
• NLO corrections to transport are large but simple
• Need a few correlators at lightlike-separated points
• Most can be extracted from the lattice
• Shear and diffusion will be harder. Stay tuned