Lattice NRQCD study of bottomonium around the deconfinement temperature

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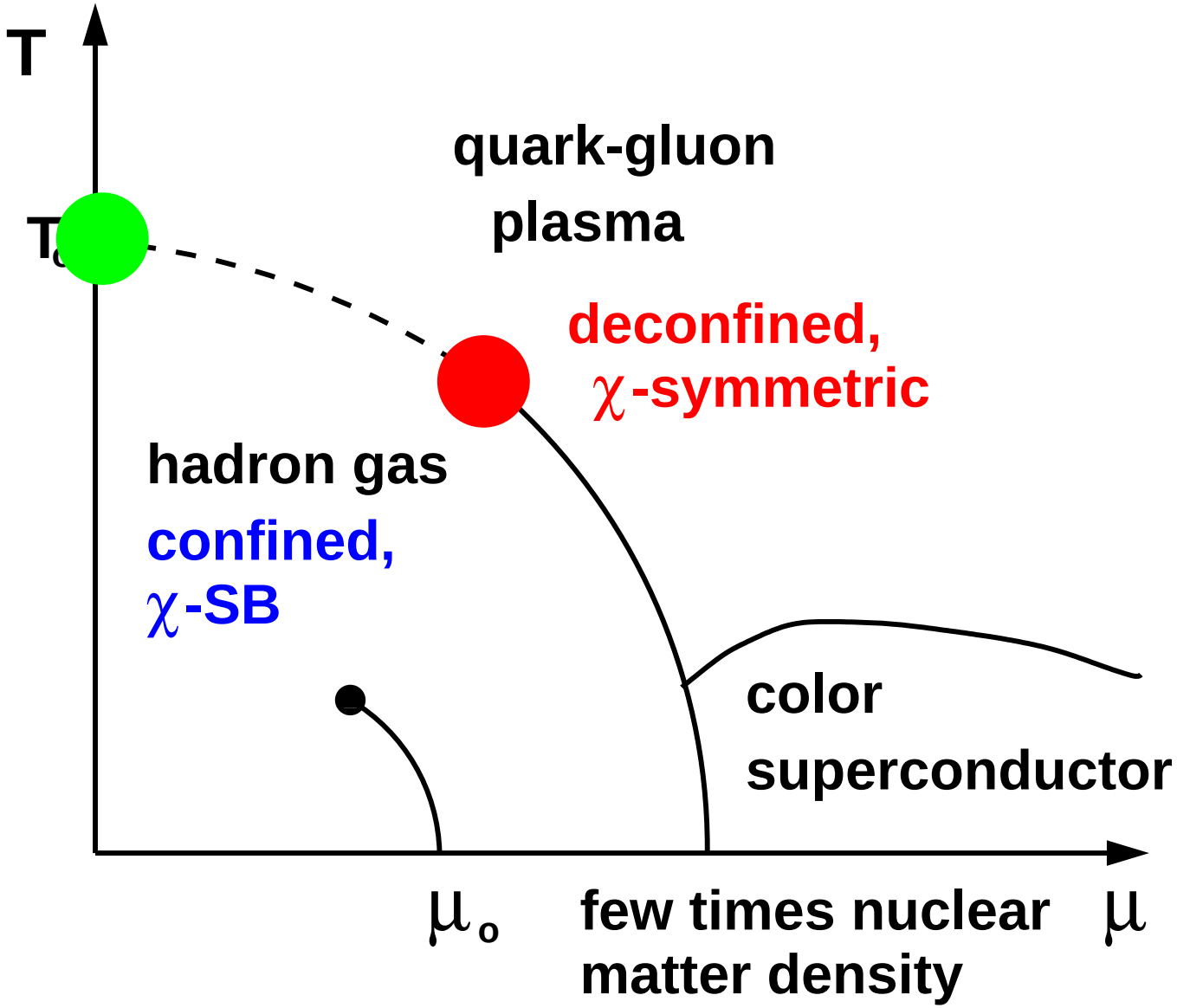
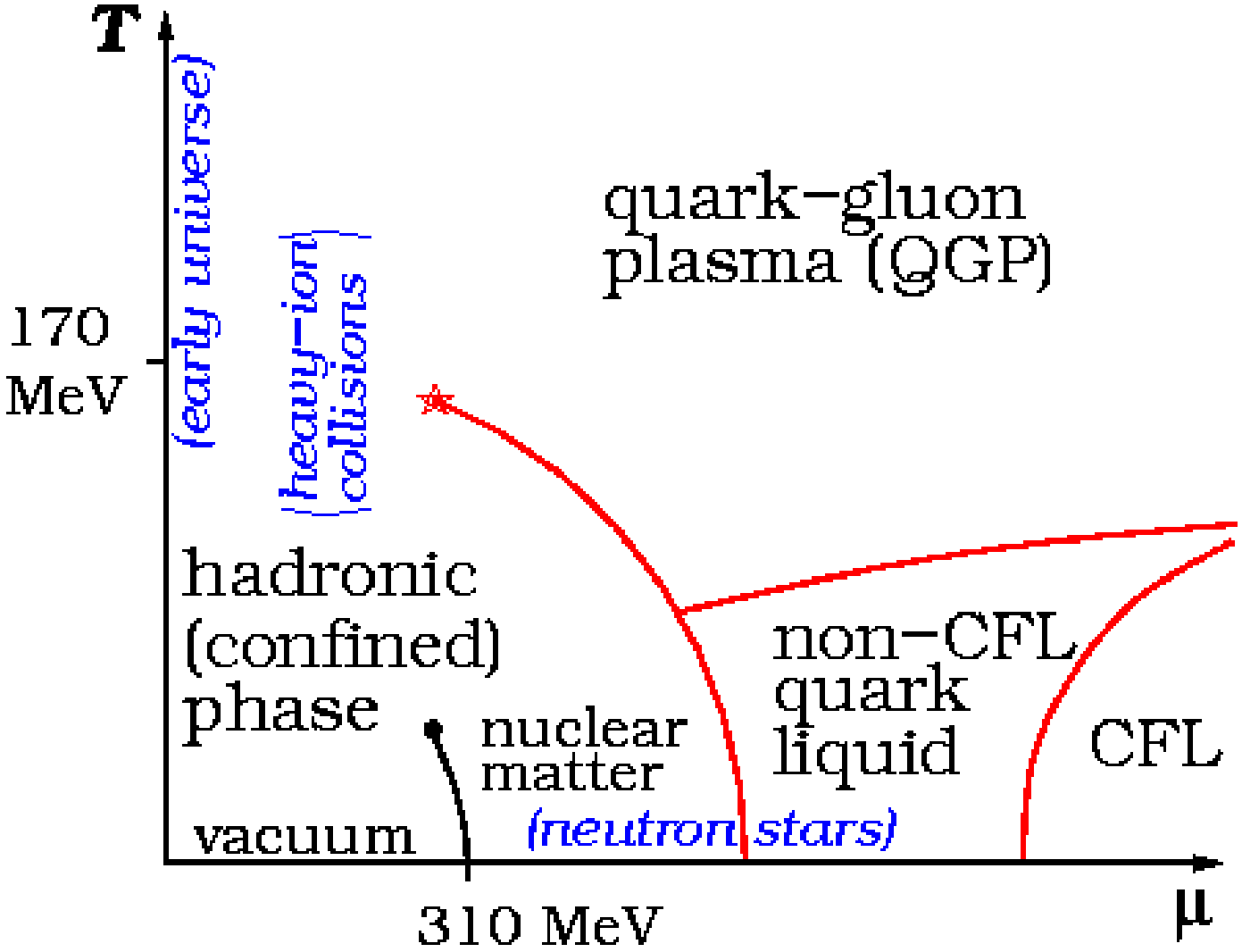
Outline

QCD Phase Diagram

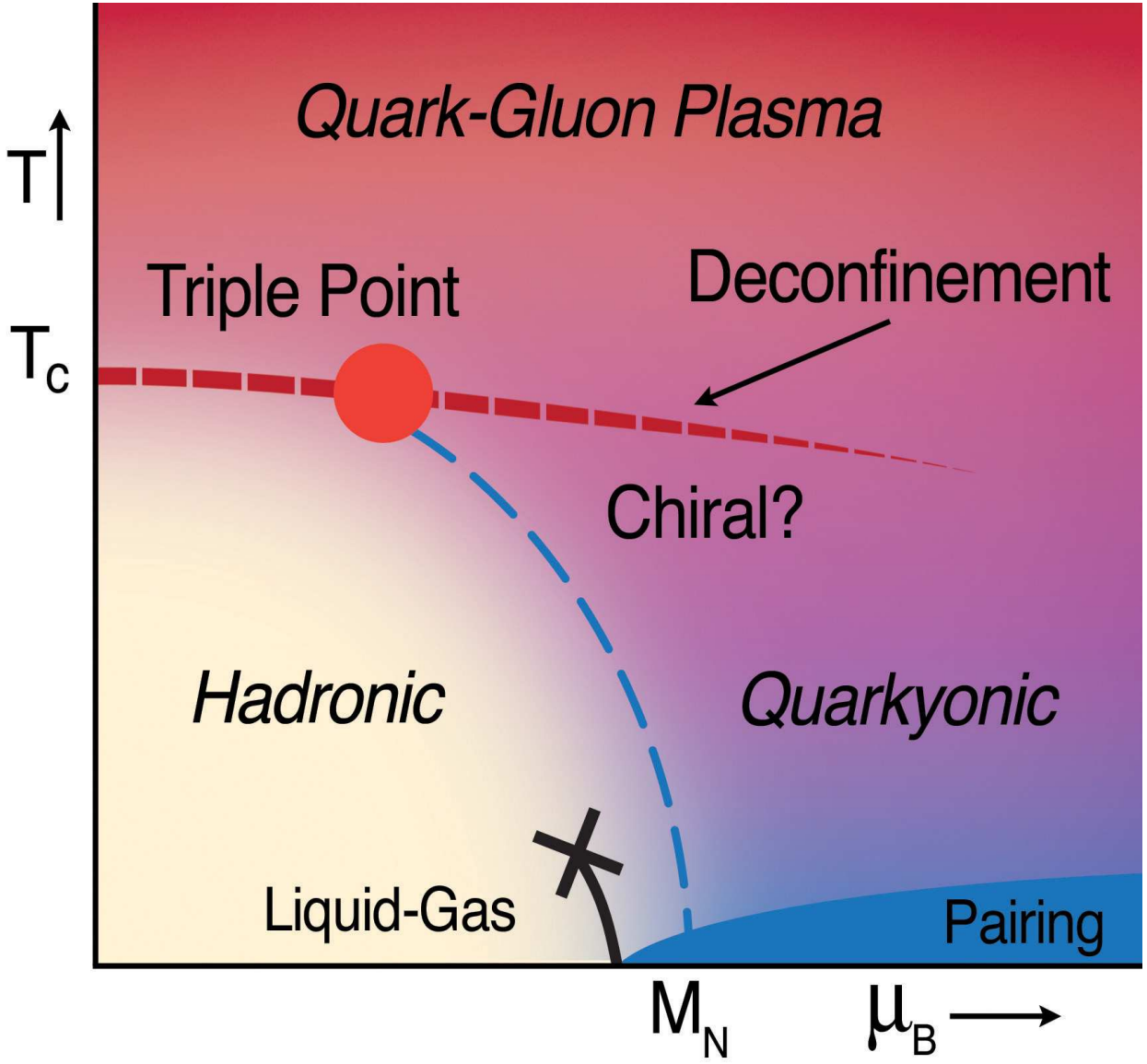


QCD Phase Diagram



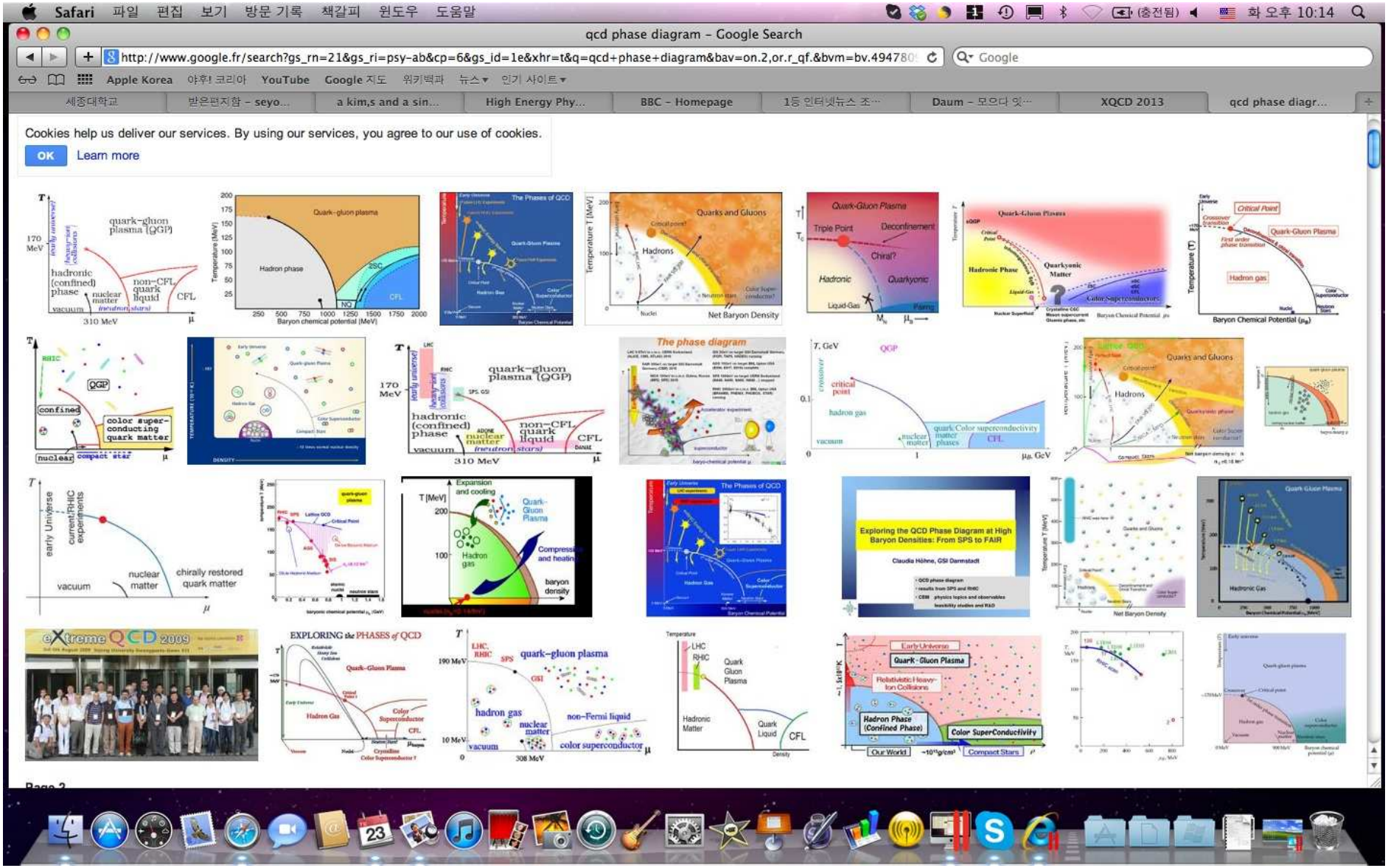
from Wikipedia

QCD Phase Diagram



from arXiv:0911.4806

QCD Phase Diagram



QCD Phase Diagram

Study of QCD phase diagram through

in-medium quarkonia behavior

Screening of colored quarks – heavy quarkonium

T. Matsui and H. Satz, PLB178 (1986) 416

Screening of colored quarks – heavy quarkonium

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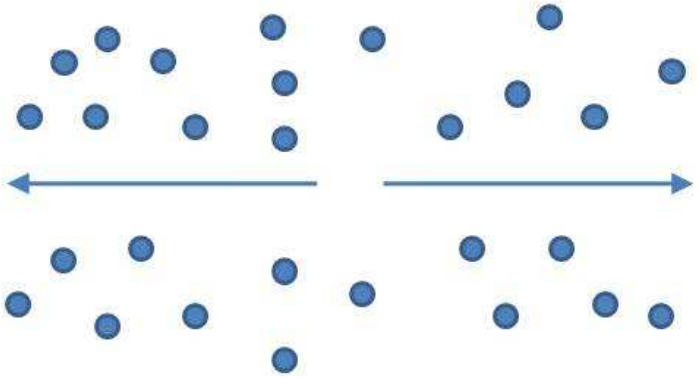


Screening of colored quarks – heavy quarkonium

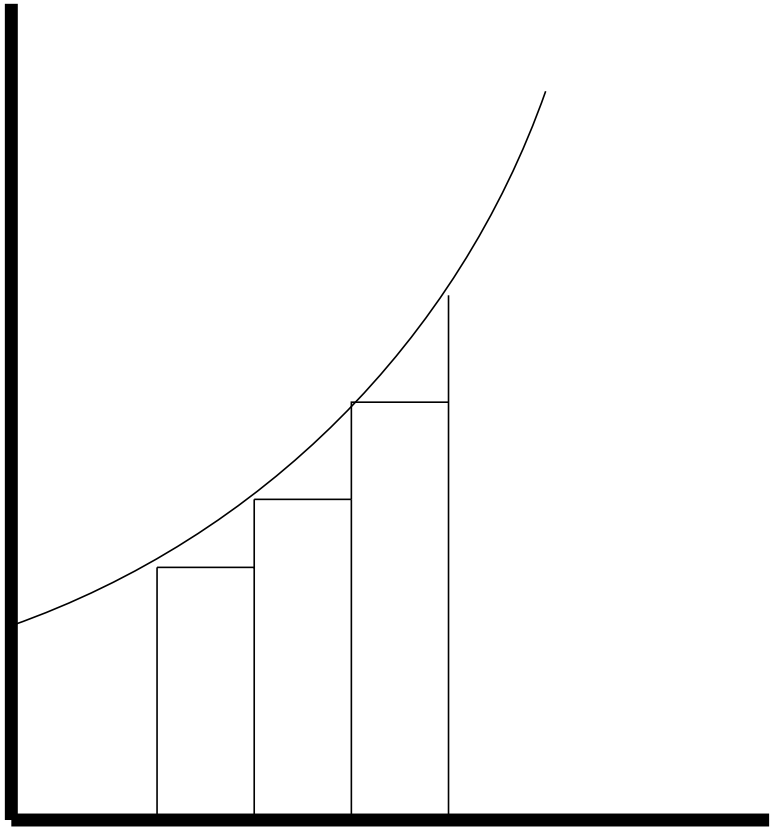
- intuitive but relies on potential model in non-zero T

- more complicated than initially thought (cold nuclear effect, regeneration, etc)

- and potential in non-zero T has imaginary part (cf. M. Laine et al, JHEP 0703 (2007) 054)

Effective field theory ($T = 0$)

- lattice theory has both UV and IR cut-off: lattice spacing (a) and lattice volume (linear dimension $N \times a$, N is the number of lattice sites)



Effective field theory ($T = 0$)

- for quarkonium:
  - ☐ size is $\sim$ 1fm
  - ☐ b-quark mass is $\sim$ 4.5GeV ($\rightarrow \lambda \sim$ 0.04fm)
  - ☐ Ma should be small to have small discretization error (say, $\sim$ 0.1)
  - ☐ $\rightarrow a =$ 0.005fm, $N = 200$
  - ☐ i.e., lattice volume needs to be as big as $200^3 \times 400$
- lattice study of quarkonium needs an idea!

Effective field theory ($T = 0$)

• NRQCD factorization for quarkonium decay/production in zero T (G.T.Bodwin, E.Bratten, G.P.Lepage, PRD51 (1995) 1125):

inclusive decay rate = partonic decay rate $\times$ the probability for heavy quark to meet anti-heavy quark

• heavy quark in the quarkonium rest frame moves slowly

□ $\rightarrow$ the heavy quark velocity, v, is a small parameter(non-relativistic)

□ $M >> Mv >> Mv^2$

□ $\frac{1}{M}$: heavy quark Compton wavelength, $\frac{1}{Mv}$: quarkonium size

• where $p > M$ is "integrated away", NRQCD

• where $p > Mv$ is "integrated away", pNRQCD (N. Brambilla, A. Pineda, J. Soto, A. Vairo, Rev.Mod.Phys. 77 (2005), 1423)

Effective field theory ($T = 0$)

• In NRQCD, physics around the scale M can be treated as point-like interactions (with effective couplings) and is separated from "bound state problem"

• NRQCD is a field theory and doesn't need to assume "potential" although "potential model" is still phenomenologically accurate

• because gluon exchange (momentum $\sim Mv$) between heavy quark (kinetic energy $\sim Mv^2$) and anti-heavy quark is "instaneous"

gluon interaction time is shorter than the scale of heavy quark motion by $1/v \rightarrow$ potential-like

• NRQCD lagrangian

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi + \delta\mathcal{L} \qquad (1)$$

Effective field theory ($T = 0$)

- In NRQCD, Lorentz symmetry is broken and power counting is complicated (cf. A. Manohar, PRD56 (1997) 167)

$$\int \frac{dE}{2\pi} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E - \frac{\mathbf{p}^2}{2M}} \tag{1}$$

- In NRQCD, mixing of soft-mode(Mv) and ultra-soft mode(Mv^2)

- In pNRQCD, "potential" is a matching coefficient between NRQCD and pNRQCD

- "Bohr radius" (size of quarkonium) enters. Multipole expansion

- pNRQCD lagrangian

$$\mathcal{L}_{\mathrm{pNRQCD}} = \Phi(\mathbf{r})^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{2M} - V^{(0)}(r) + \delta V + \cdots \right) \Phi(\mathbf{r}) \tag{2}$$

Effective field theory ($T \neq 0$)

- quarkonium in non-zero T has two scales: heavy quark mass (M) and temperature (T)

- depending on the relative magnitudes of M, Mv, Mv^2 and T, gT, g^2T, careful consideration is necessary

- strongly coupled (sQCD) around a few T_c (cf. B. Muller, Acta. Phys. Polon. B38 (2007) 3705)

heavy quark propagator in NRQCD

- heavy quark propagator in QCD

$$(\gamma_\mu D_\mu + m)G(\mathbf{x}, \tau) = S(\mathbf{x})\delta_{t,0} \tag{3}$$

boundary value problem

- heavy quark propagator in NRQCD

$$(D_t - H_0 - \delta H)G(\mathbf{x}, \tau) = S(\mathbf{x})\delta_{t,0} \tag{4}$$

initial value problem

- quarkonium propagator

$$G_{\text{meson}}(\tau) = \sum_{\mathbf{x}} G^\dagger_{ab}(\mathbf{x}, \tau) G_{cd}(\mathbf{x}, \tau) \tag{5}$$

- gauge field and light quark have “temperature effect”

spectral function in NRQCD

In QCD,

$$G_\Gamma(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau, \vec{x}) \Gamma \psi(\tau, \vec{x}) \overline{\psi}(0, \vec{0}) \Gamma \psi(0, \vec{0}) \rangle \tag{6}$$

$$= \int \frac{d^3 p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho_\Gamma(\omega, \vec{p}) \tag{7}$$

and

$$K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \tag{8}$$

- the spectral function of Euclidean correlator has all the information on the finite temperature behavior of a propagator

- numerically ill-posed problem

- Maximum Entropy Method is used (cf. M. Asakawa, T. Hatsuda, Y. Nakahara, PPNP46 (2001) 459)

spectral function in NRQCD

- known to have problems (cf. T. Umeda, PRD75 (2007) 094502 and A. Mocsy and P. Petreczky, PRD77 (2008) 014501)

- both the kernel($K(\tau, \omega)$) and the spectral density($\rho_\Gamma(\omega, \vec{p})$) depend on temperature

- constant contribution

- In NRQCD, with $\omega = 2M + \omega'$ and $T/M << 1$, $K(\tau, \omega) \to e^{-\omega\tau}$

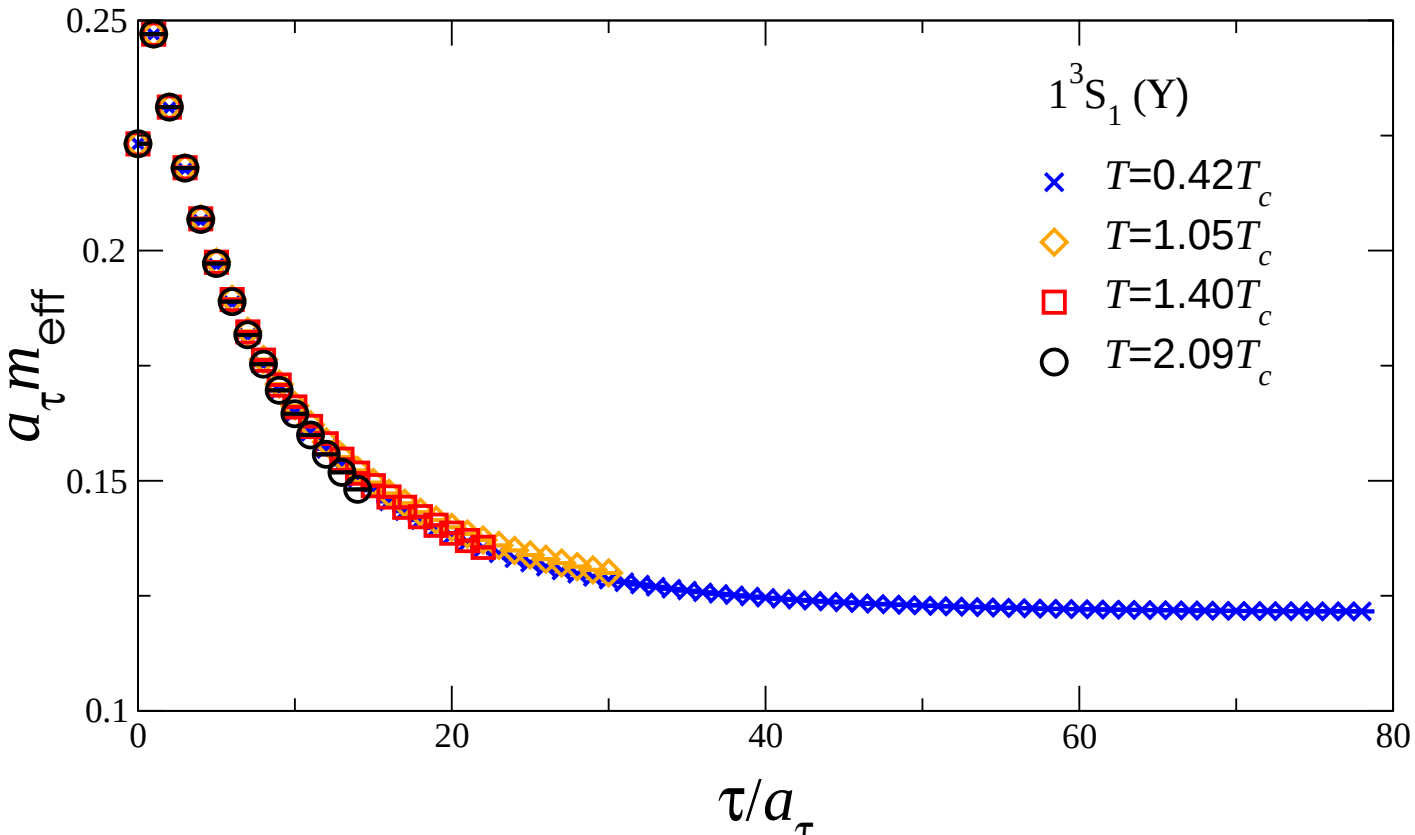
$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega') \tag{6}$$

- numerical inverse Laplace transform problem

S-wave bottomonium at $T \neq 0$

G. Aarts, S.K., M.P.Lombardo, M.B. Oktay, S.M. Ryan, D.K. Sinclair, J.I. Skullerud, PRL106 (2011) 061602

- bound state $\rightarrow$ exponentially falling propagator ($G(\tau) \sim Ae^{-E\tau}$)
- $m_{\text{eff}}(\tau) = -\log[G(\tau)/G(\tau - a_\tau)] \rightarrow$ constant



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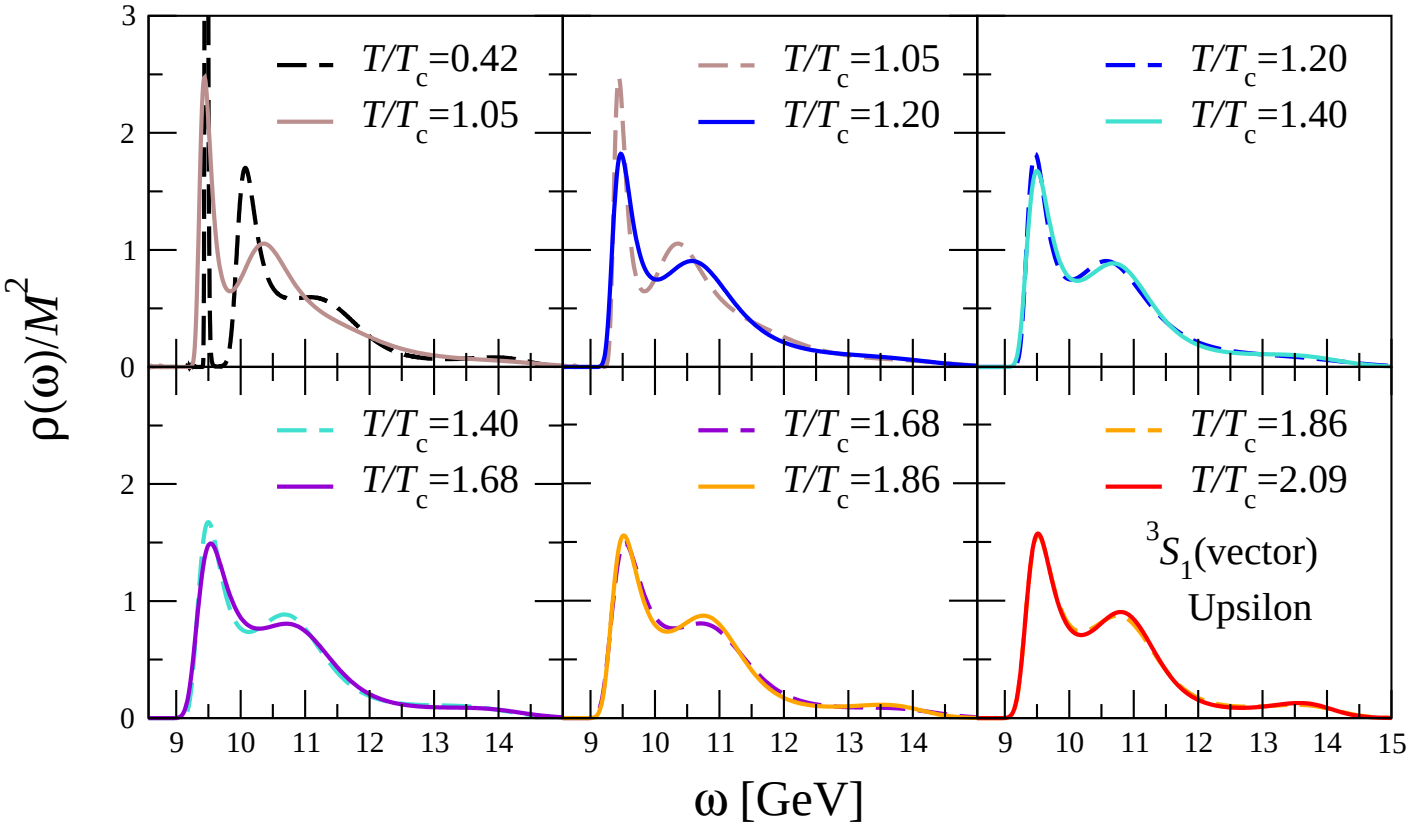
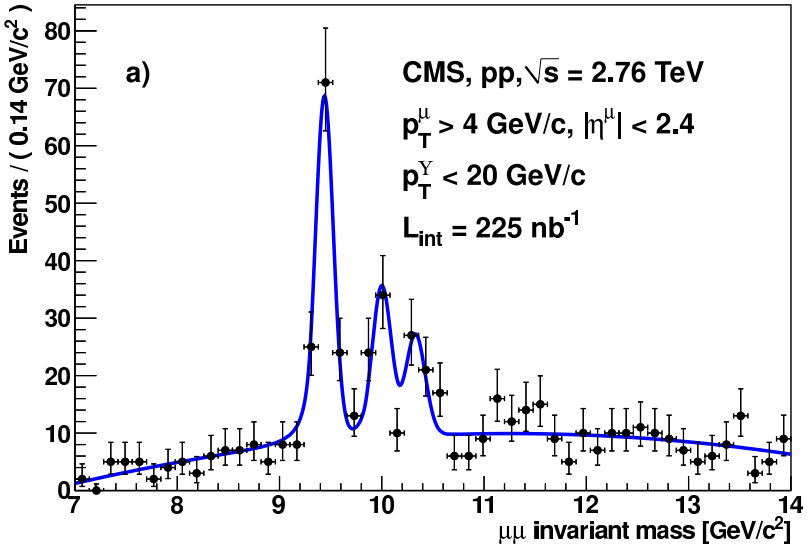
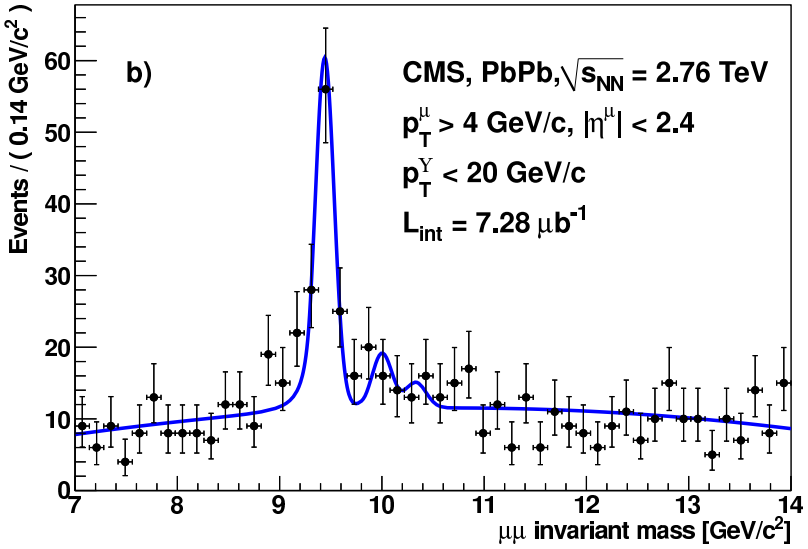
G. Aarts, C. Allton, S.K., M.P.Lombardo, M.B. Oktay, S.M. Ryan, D.K. Sinclair, J.I. Skullerud, JHEP11 (2011) 103



- spectral function of Upsilon with lattice NRQCD kernel ($K(\tau) = e^{-\omega\tau}$)
- $1S$ peak survives, $2S$ and higher peaks merge and become a broad peak as T increases (melting)

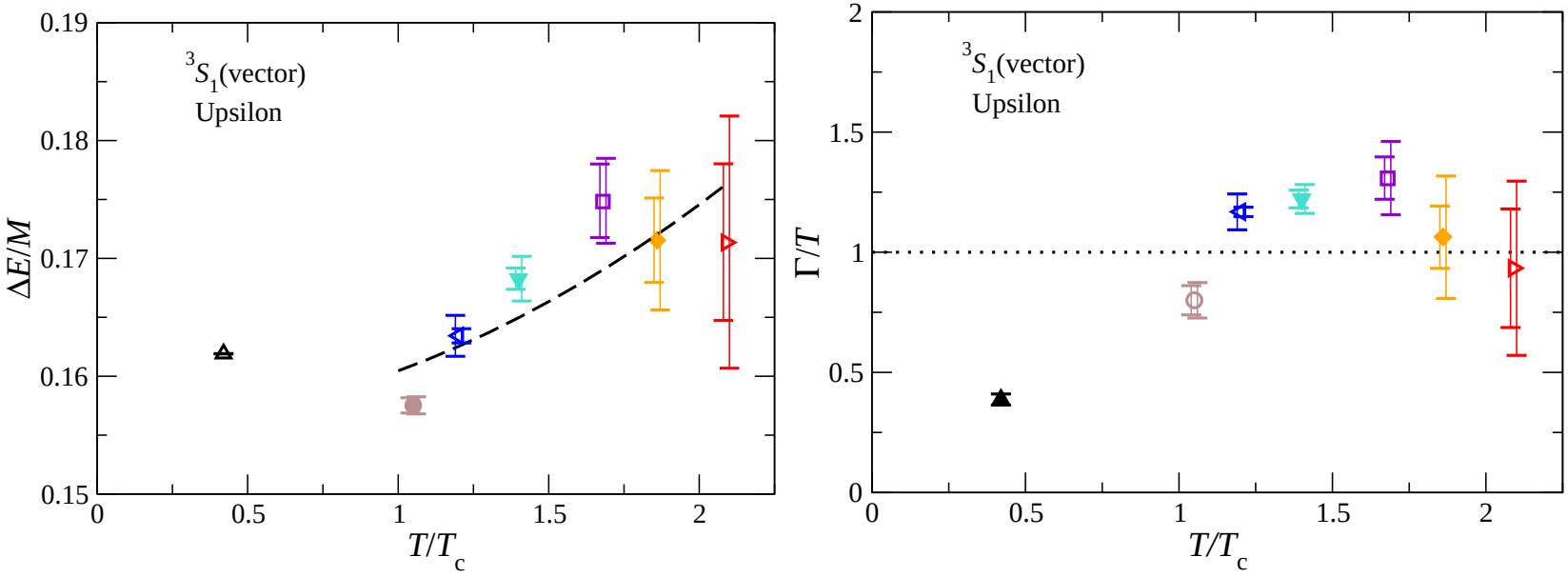
CMS collaboration, PRL107 (2011) 052302





- In 2011, CMS collaboration observed disappearance of 2S and 3S upsilon state in Pb-Pb collisions
- sequential suppression

T-dependence of the Υ ground state peak



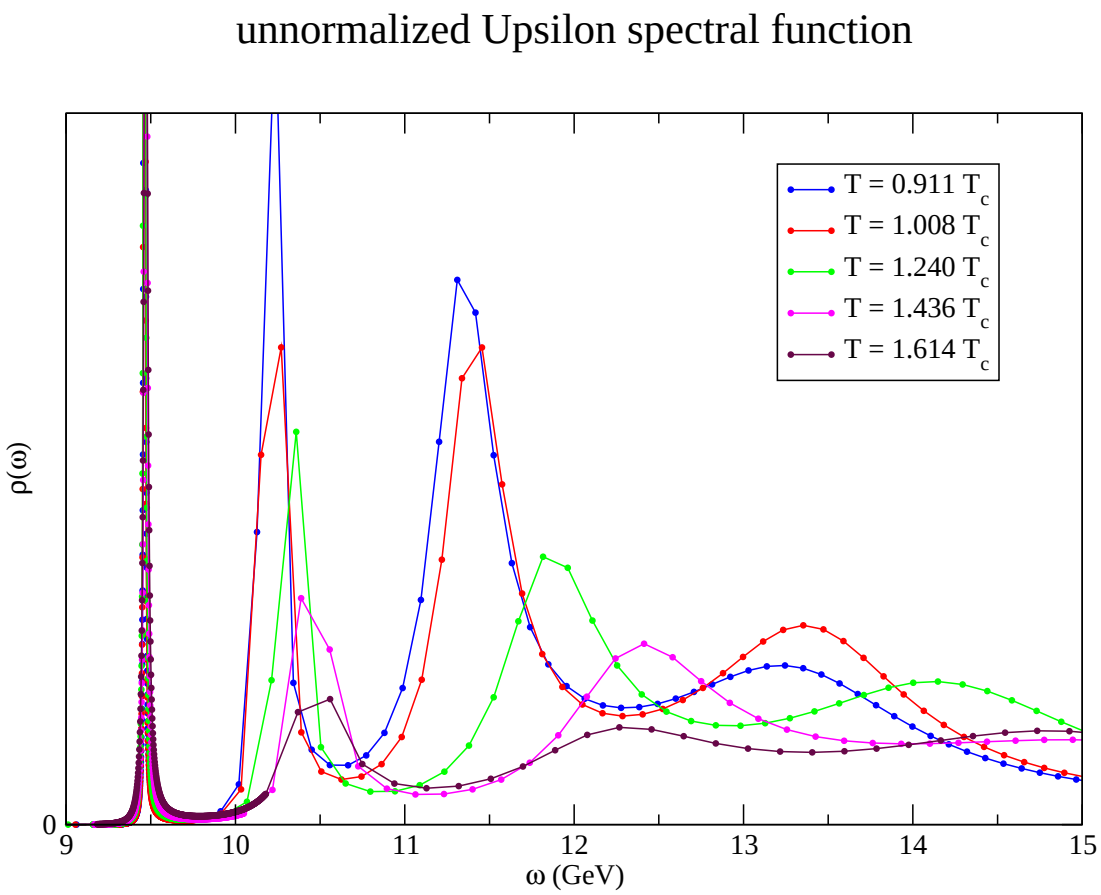
$$\Delta E \sim \alpha_s \frac{T^2}{M}, \qquad \frac{\Gamma}{T} \sim \alpha_s^3 \quad (\alpha_s \sim 0.4) \tag{7}$$

(N.Brambilla, M.A.Escobedo, J. Ghiglieri, J. Soto, A. Vairo, JHEP1009 (2010) 038)

- dissociation may be due to broadening of thermal width not vanishing binding energy (N. Brambilla et al)

T-dependence of the Υ ground state peak

S.K, P.P, A.R., on $48^3 \times 12, N_f = 2+1$(HiSQ) HotQCD lattice (Lat2013)



- improved Bayesian method (A. Rothkopf, Lat2013 and, AR and Y. Burinner, arXiv:1307.6106)

T-dependence of the Υ ground state peak

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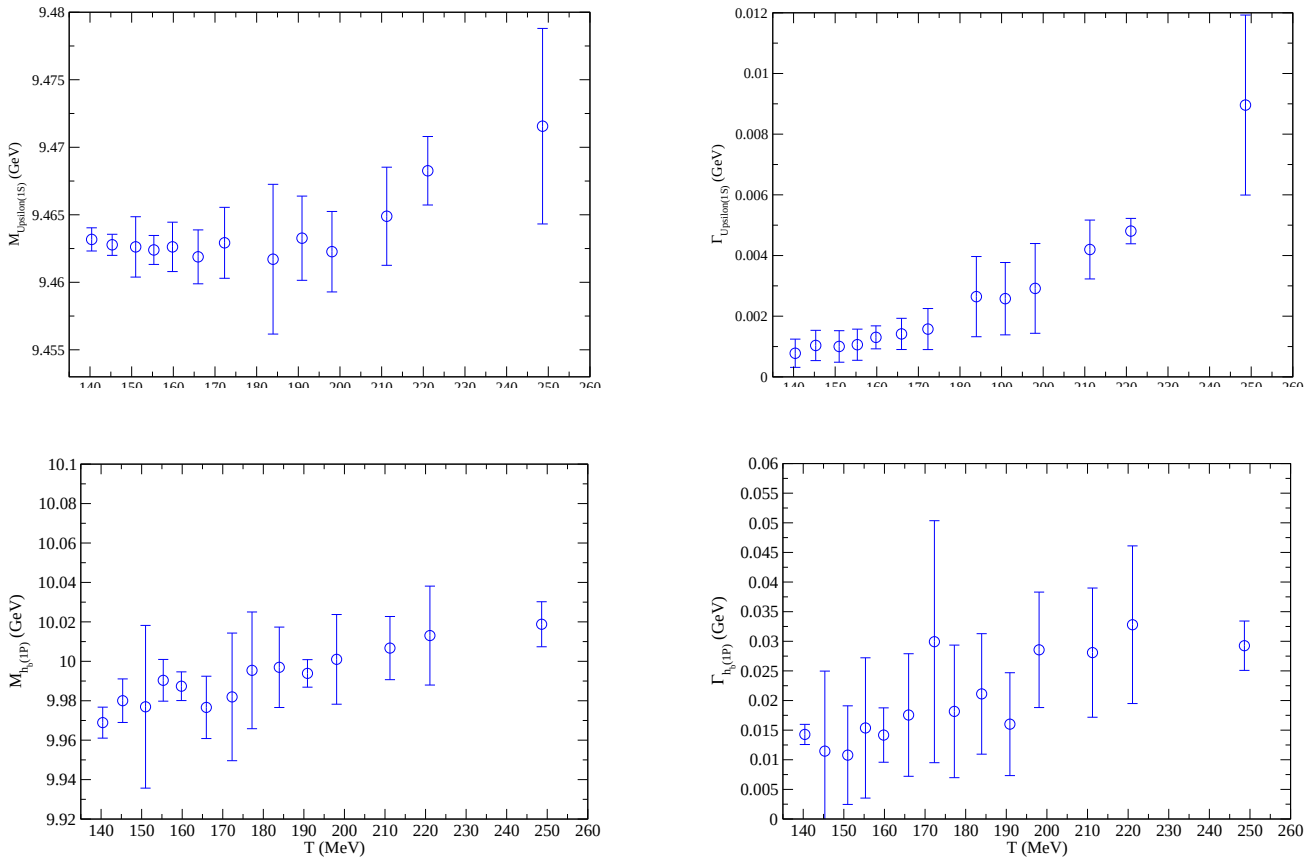
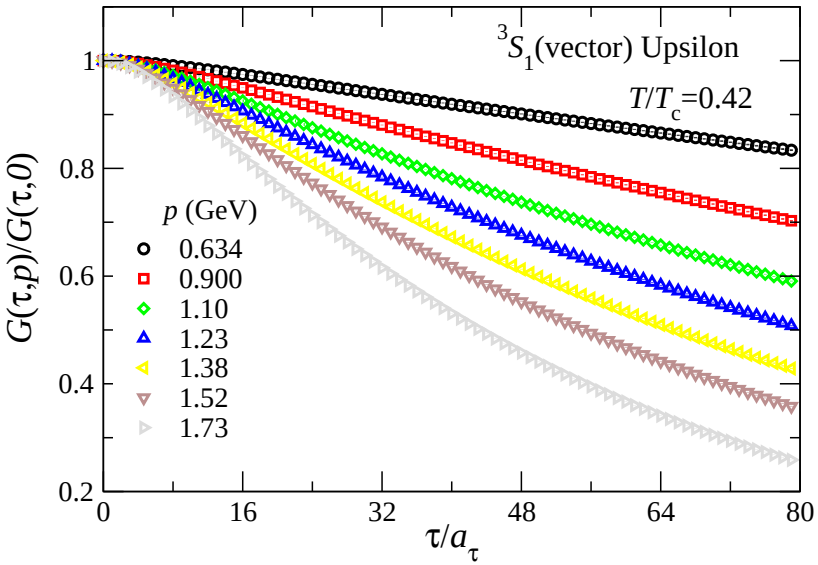
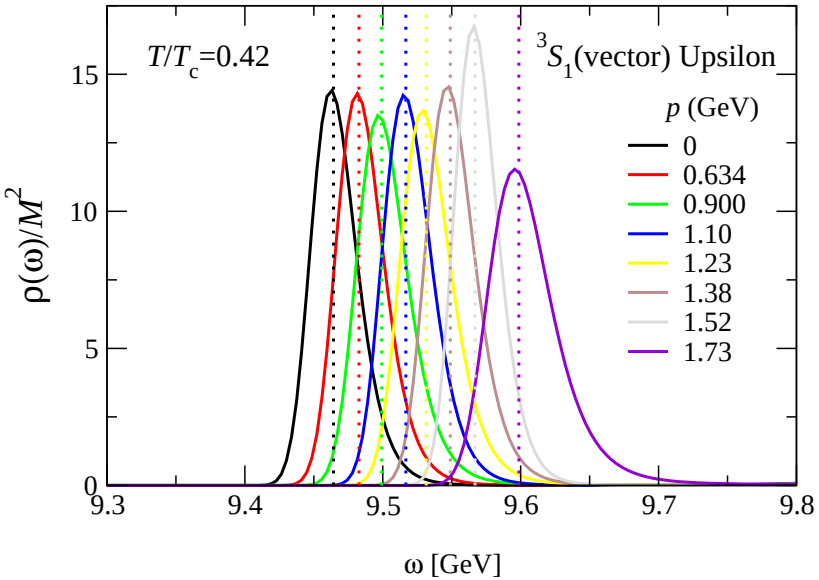


S-wave moving in thermal bath

G. Aarts et al, JHEP1303 (2013) 084, T. Harris Lat2013

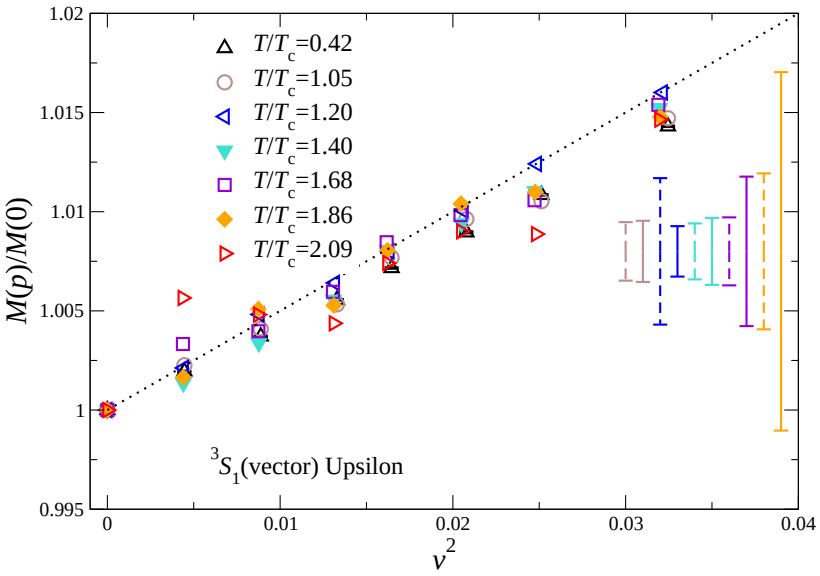
- moving with non-relativistic velocity ($\frac{v}{c} \leq 0.2$ at $T \sim 0$)

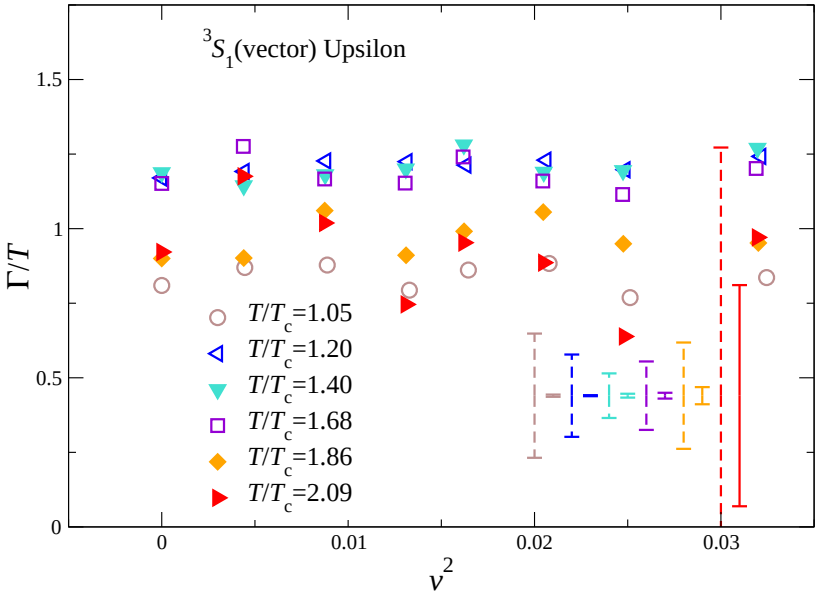




- the ratio, $G(\tau,\vec{p})/G(\tau,\vec{0})$(left) and the spectral function (right) shows momentum dependence

S-wave moving in thermal bath





- dispersion relation is satisfied but momentum dependence of thermal width is very small (or negligible) at the range of momenta studied

$$\delta E_{\mathrm{s-wave}} \sim \alpha_s \frac{T^2}{M}, \qquad \frac{\Gamma(v)}{\Gamma(v=0)} \sim \frac{1}{\sqrt{1-v^2}} \tag{7}$$

M.A.Escobedo, F.Giannuzzi, M.Mannarelli, J.Soto, PRD87 (2013) 114005

S-wave moving in thermal bath

M.A.Escobedo, F.Giannuzzi, M.Mannarelli, J.Soto, PRD87 (2013) 114005

• quarkonium may be produced in fragmentation process $\rightarrow$ quarkonium is moving with regard to thermal bath

• two possible hierachies:

$$M >> 1/r >> T >> E >> m_D(I) \quad M >> T >> 1/r >> m_D >> E(II) \tag{7}$$

• for S-wave, the energy shift is independent of velocity

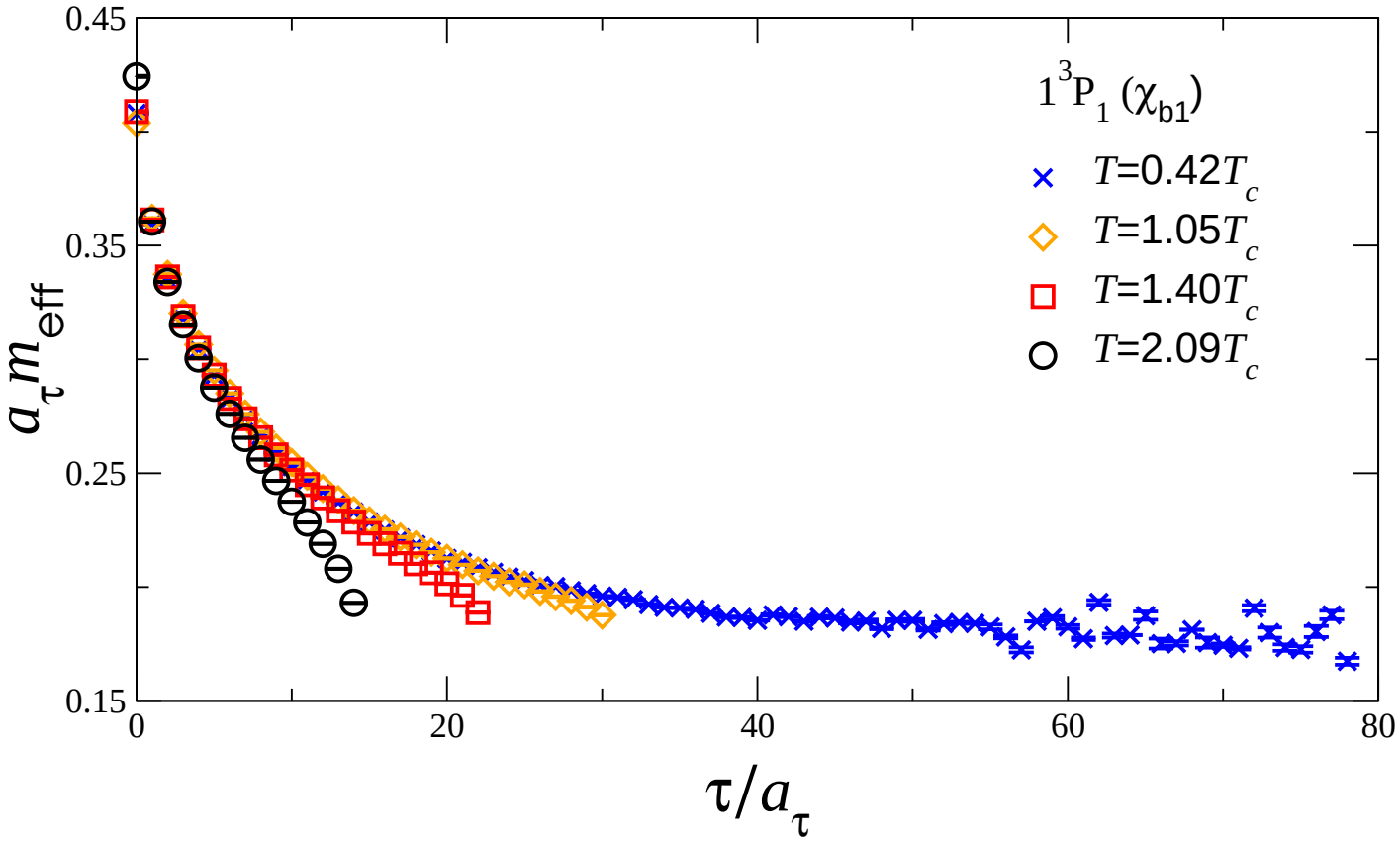
• the width depends on the velocity non-trivially

• less important for bottomonium than for charmonium

P-wave bottomonium at $T \neq 0$

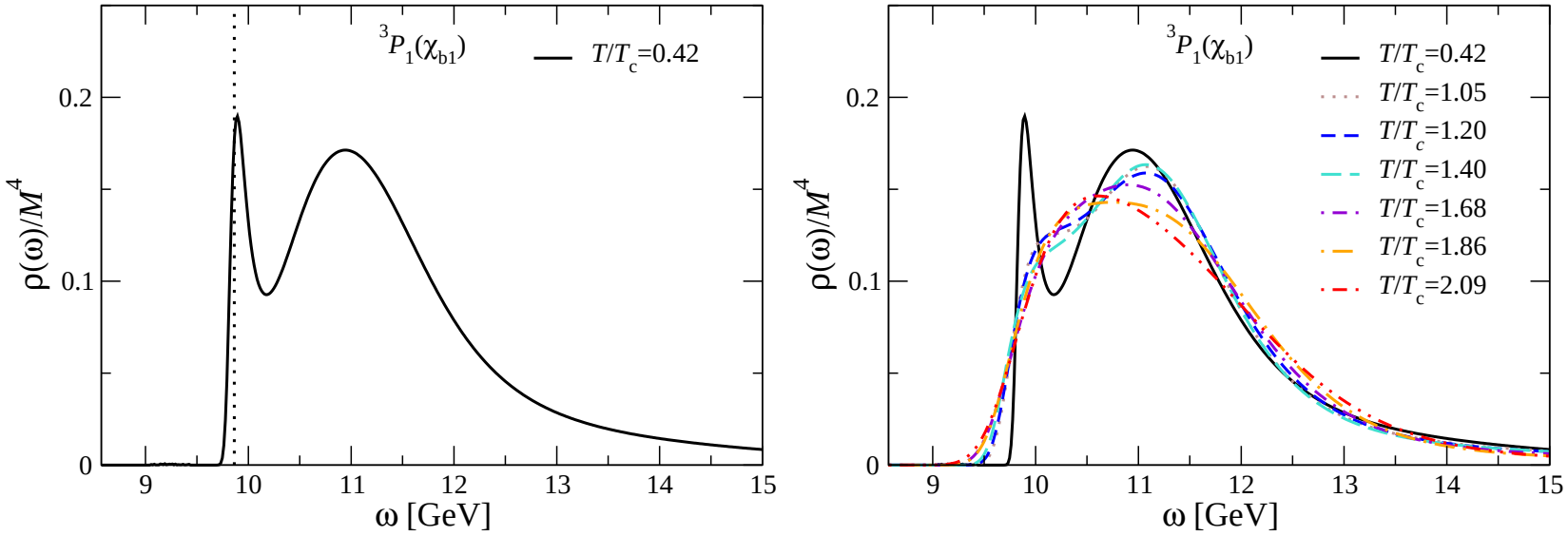
G. Aarts et al, PRL106 (2011) 061602, T. Harris Lat2013

- non-bound state $\rightarrow$ power-like propagator ($G(\tau) \sim A'\tau^{-\gamma}$) in $\sim 2T_c$
- $m_{\text{eff}}(\tau) = -\log[G(\tau)/G(\tau - a_\tau)]$



P-wave bottomonium at $T \neq 0$

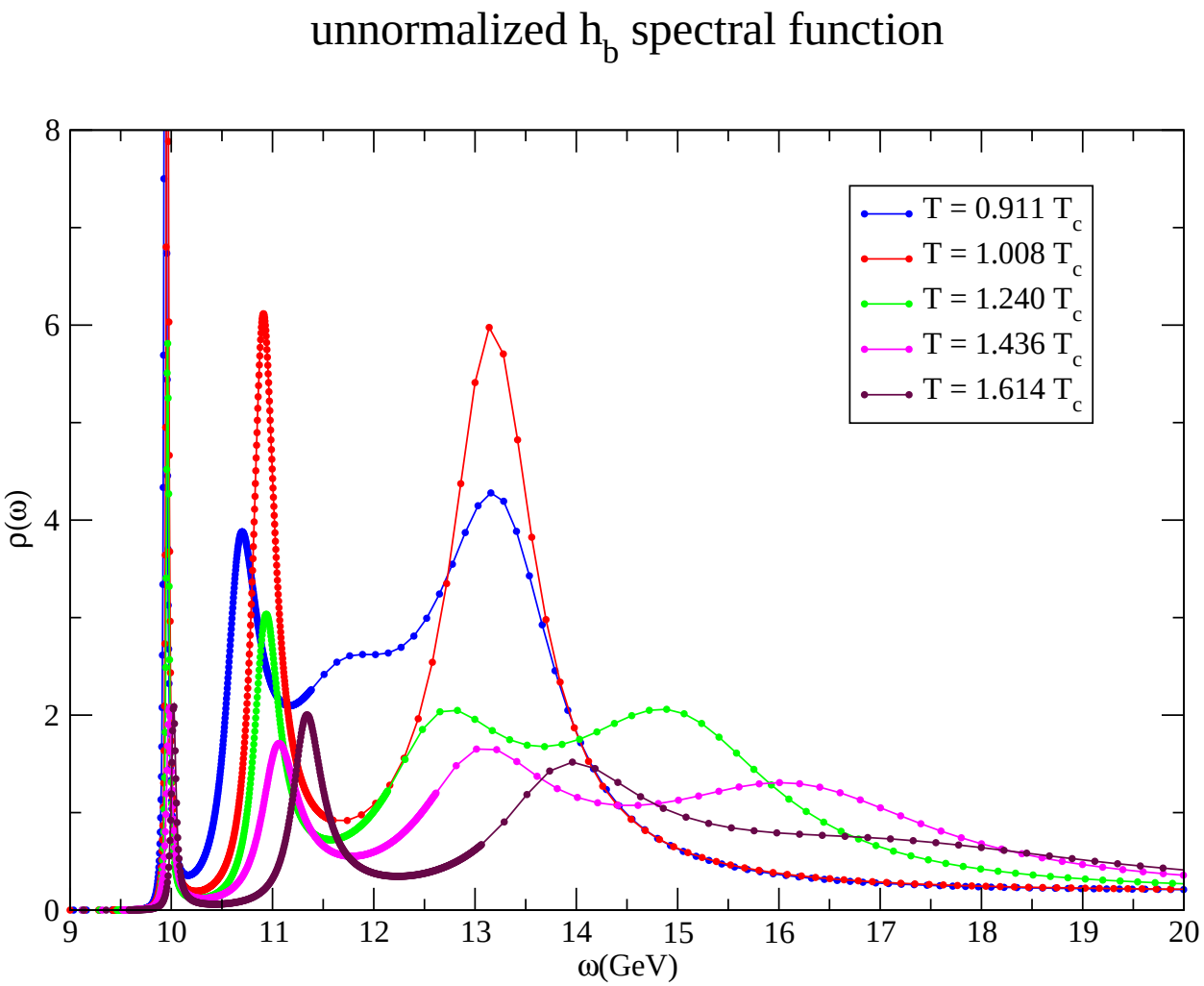
G. Aarts, Lat2013



- dotted line is experimental value for the χ_{b1}
- melts above T_c

P-wave bottomonium at $T \neq 0$

S.K, P.P, A.R., on $48^3 \times 12, N_f = 2 + 1$ HiSQ lattice (Lat2013)

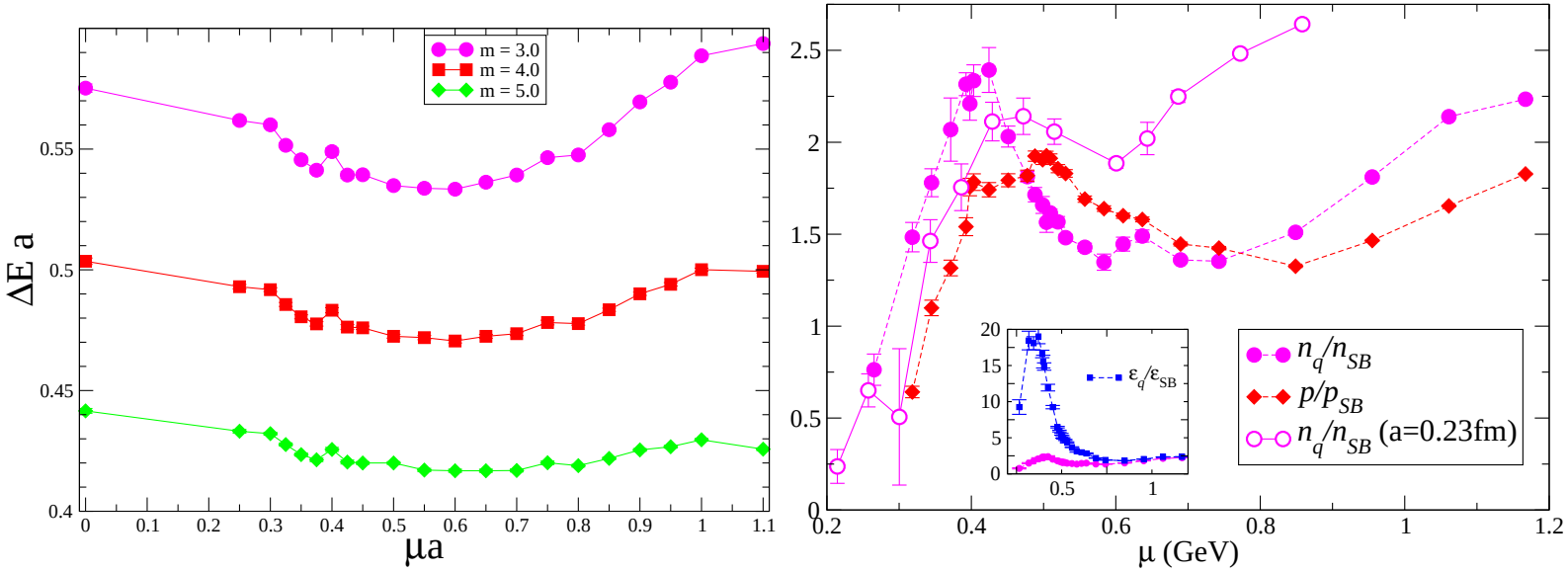


NR spectrum in QCD-like Theory

- lattice QCD $\mu \neq 0$ is "difficult" (complex action problem)

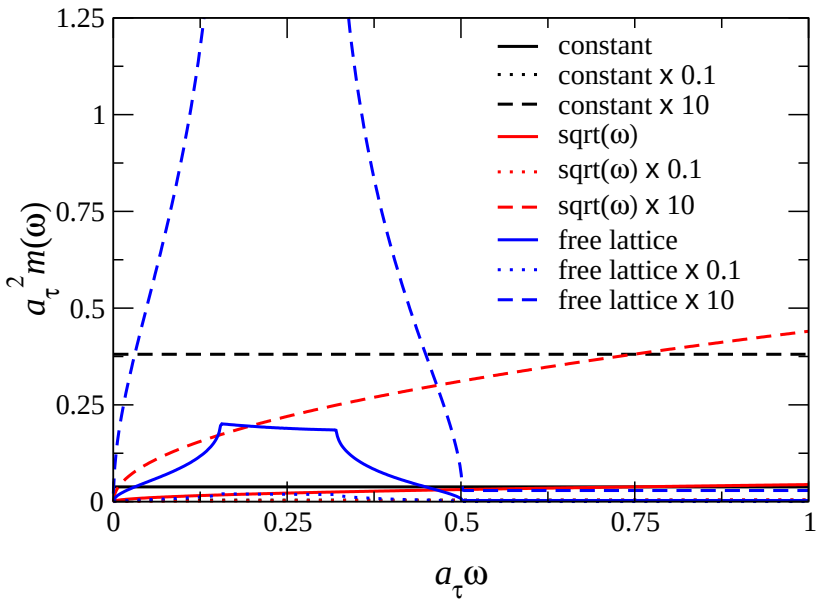
- NRQCD with isospin chemical potential (cf. W. Detmold's talk? or W. Detmold, et al, PRD87 (2013) 094504)

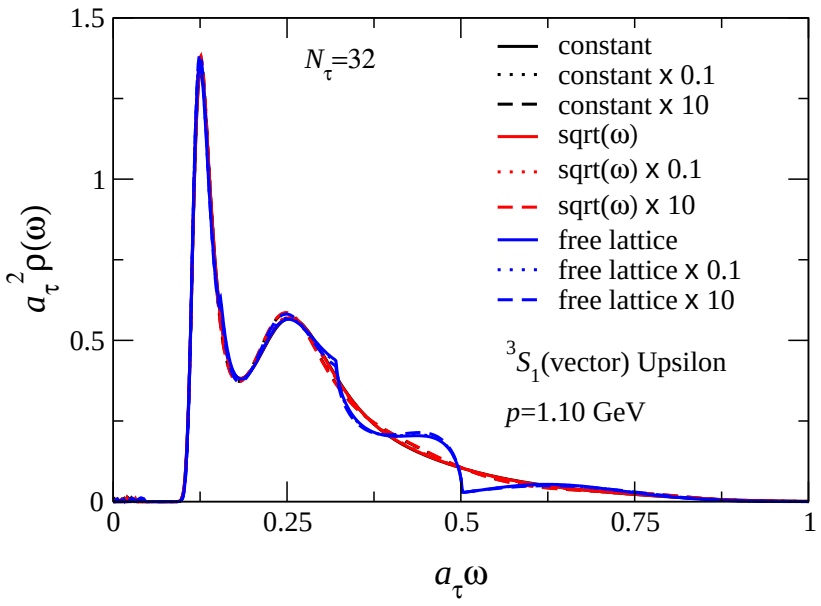
- NRQC$_2$D, $SU(2)$ gauge theory with baryon chemical potential (cf. S. Hands, S.K., J.-I. Skullerud, PLB711 (2012) 199)



Need more study for P-wave ?

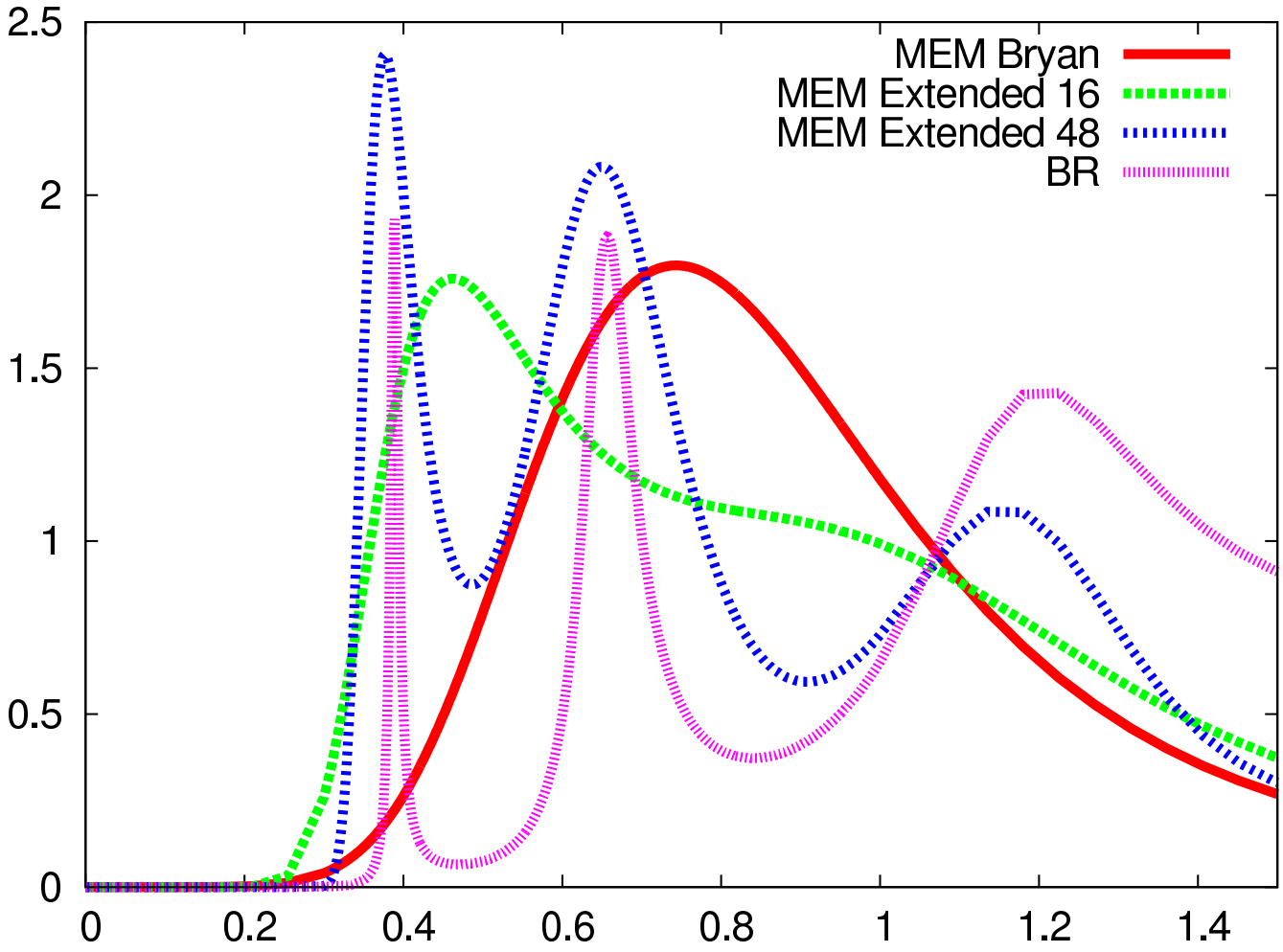
- G. Aarts Trento2013, default model independence





Need more study for P-wave ?

- S.K, A. Rothkopf (preliminary), improved Bayesian method



Conclusion

• Effective field theories for quarkonium such as NRQCD and pNRQCD offers an opportunity to study quarkonia quantitatively at the temperature near the deconfinement temperature

• Lattice NRQCD at non-zero temperature allows us to study non-perturbative finite temperature behavior of quarkonium without relying on “heavy quark potential

• Above the deconfinement temperature, S-wave quarkonium still show a bound state behavior upto $\sim 2T_c$. (1S) peak of S-wave quarkonium (upsilon, eta_b) persists in this temperature range but (2S) and (3S) are reduced/merged

• P-wave melts in high temperature but more study is required around T_c