Scale hierarchy in high-temperature QCD

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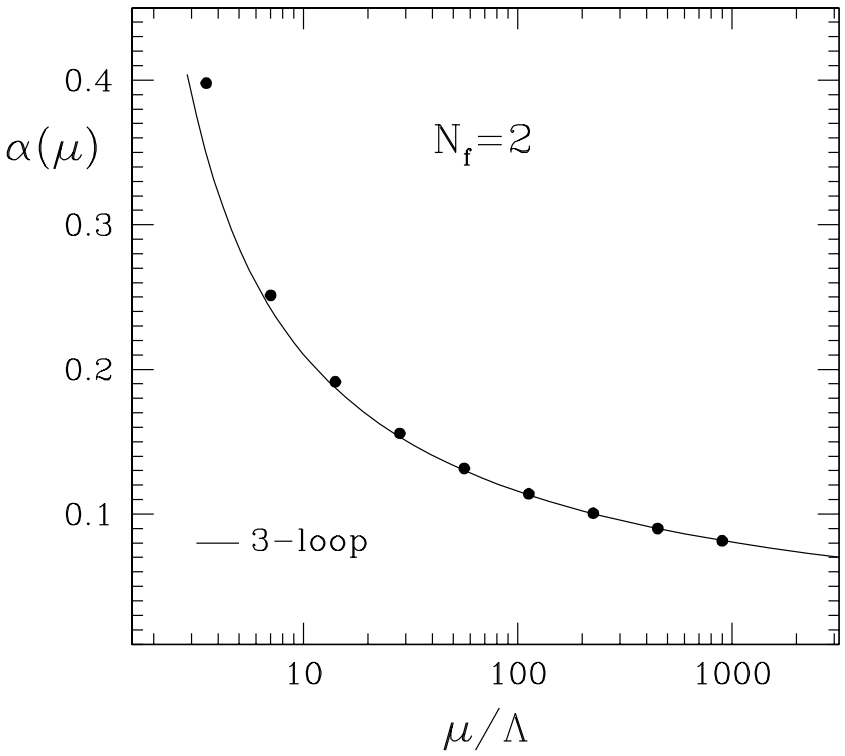
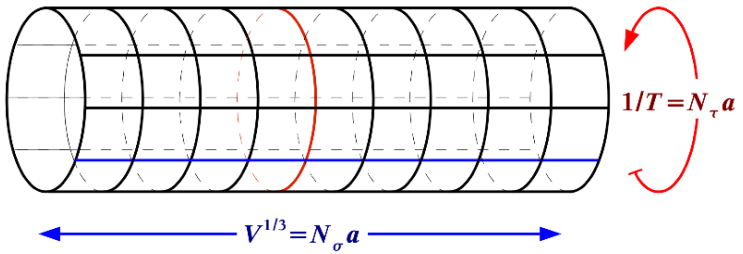
QCD is asymptotically free



- High temperature: $g(T) \to 0$, deconfinement for $T > T_c$
- Perturbative treatment OK for “sufficiently high" T

Dimensional reduction: $4d \to 3d$

- Thermal boundary conditions:
  $\phi(x + 1/T) = \pm\phi(x)$ bosons / fermions

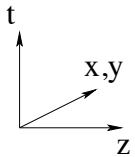


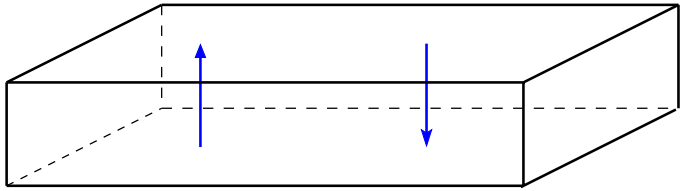
- Fourier decomposition: $\tilde{\phi}_n(x) = \int_0^{1/T} \mathrm{d}t\, e^{i2\pi(n+q)t}\phi(x,t)$ $q = \{0, 1/2\}$
- Tower of states: $E_n^2 = |\vec{k}|^2 + (2\pi T(n+q))^2 + m^2 = |\vec{k}|^2 + \left(m_{\text{eff}}^{3d}\right)^2$
- $|\vec{k}| \ll T \to$ static $(n = 0)$ modes for bosons, fermions decouple

Same for gauge fields

- Effective d.o.f: $\bar{A}_i \equiv A_{i,n=0}$ and $\bar{A}_0 \equiv A_{0,n=0}$ or Polyakov loop L

$$\bar{A}_0 \equiv A_{0,n=0}(\vec{x}) = \int_0^{1/T} \mathrm{d}t\, A_0^a(\vec{x},t)\tau_a$$





- Effective action:
$$S^{4d} = \int \mathrm{d}^3x \mathrm{d}t\, \mathrm{Tr} F_{\mu\nu}^2 \to S_{\text{eff}}^{3d} = \int \mathrm{d}^3x \left[\mathrm{Tr}\bar{F}_{ij}^2 + m^2 \bar{A}_0^2 + \left(D_i \bar{A}_0\right)^2 + \lambda \bar{A}_0^4 + \cdots \right]$$
i.e. $3d$ Yang-Mills + adjoint Higgs

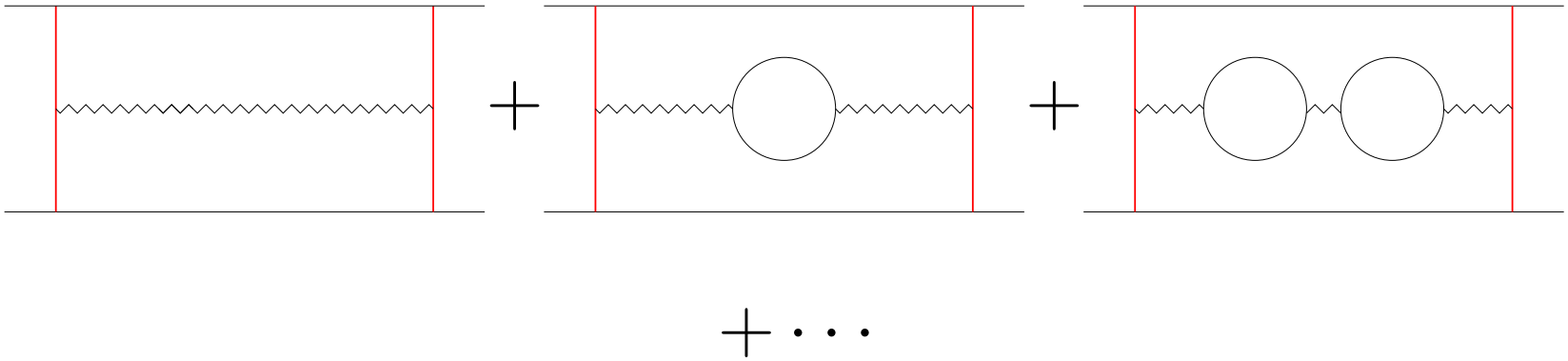
$3d$ coupling by integrating our non-static modes.
Tree level: $\left(g_{\text{eff}}^{3d}\right)^2 = g(T)T$.

Note: $3d$ theory is confining, i.e. non-perturbative in IR (spatial string tension, glueball.. . .)

Linde: non-perturbative scale is $g^2(T)T$

Debye screening

- QED: e^+e^- thermal pair creation screens static charges

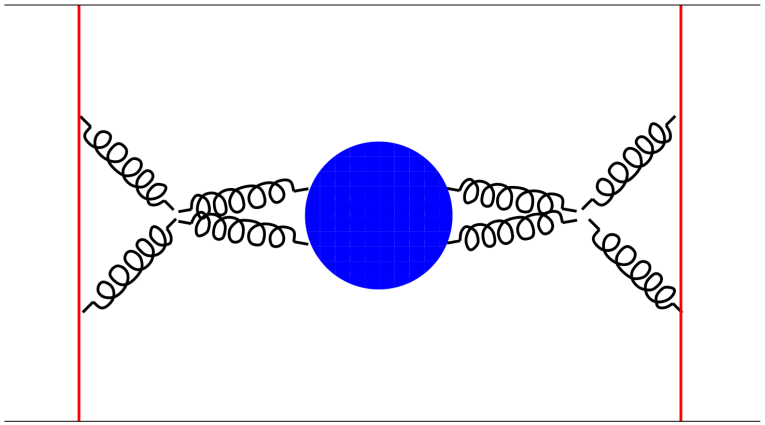


- Screening: $m_E^2 = -\Pi^{00}(k_0 = 0, \vec{k} \to \vec{0})$, $\boxed{m_E = \frac{eT}{\sqrt{3}}\left(1 + \mathcal{O}\left(e^2\right)\right)}$
- Coulomb potential $\propto r^{-1}$ at $T = 0 \to$ Yukawa $\propto \frac{\exp(-m_E r)}{r}$ at $T > 0$

Debye screening: QCD

- QCD: Tr(Polyakov loop) is color singlet, so at least two gluons emitted
- To lowest order, similar to QED: $m_E = gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$
- Higher order: $4-$gluon vertex couples to $3d$ glueball:

$m_G \sim \mathcal{O}\left(g^2 T\right)$



Confining potential $\propto \sigma r$ at $T = 0 \rightarrow \propto \frac{\exp(-m_E r)}{r} \underset{r \nearrow}{\rightarrow} \propto \frac{\exp(-m_G r)}{r}$

Recap: 3 scales in high-T QCD

- "hard" scale $2\pi T$ non-static modes
- "soft" scale $g(T)T$ Debye mass ("electric")
- "ultrasoft" scale $g^2(T)T$ $3d$ glueball mass ("magnetic")

Hierarchy as $T \to \infty$, $g(T) \to 0$

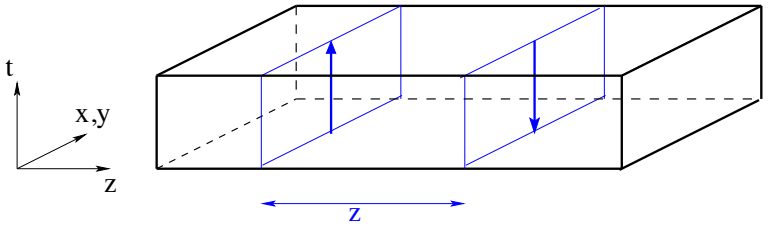
- integrate out "hard" scale $\to$ effective theory EQCD (electric)
  $3d$ Yang-Mills + adjoint Higgs
- integrate out "soft" scale $\to$ effective theory MQCD (magnetic)
  $3d$ Yang-Mills

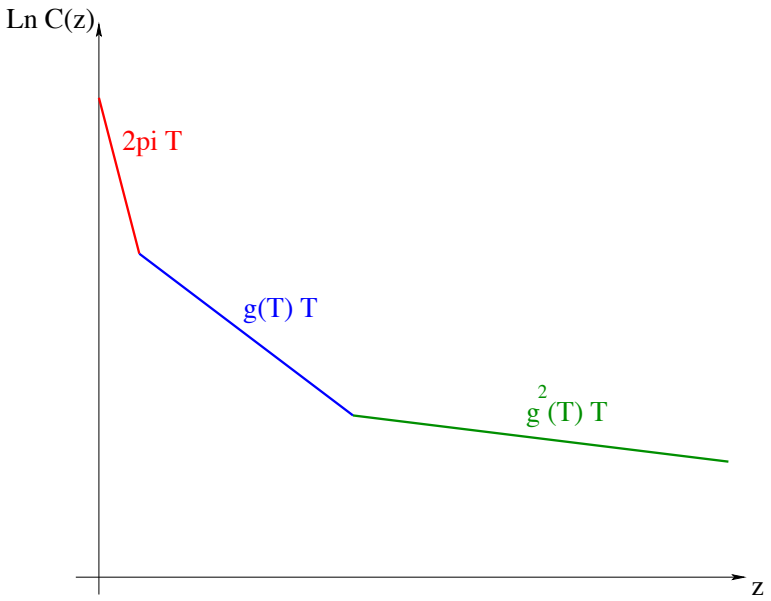
..., Kajantie et al, ...

How high should T be?

$$g^2 T \ll gT \ll 2\pi T$$

Look for three scales in decay rate of correlator of Polyakov loops





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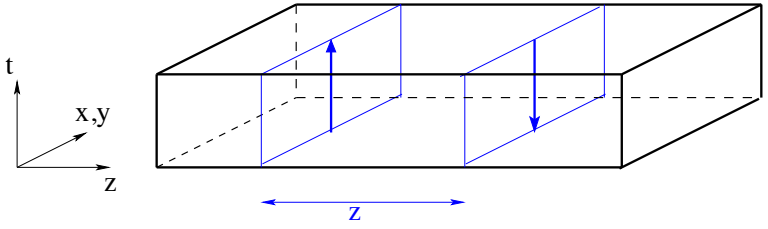
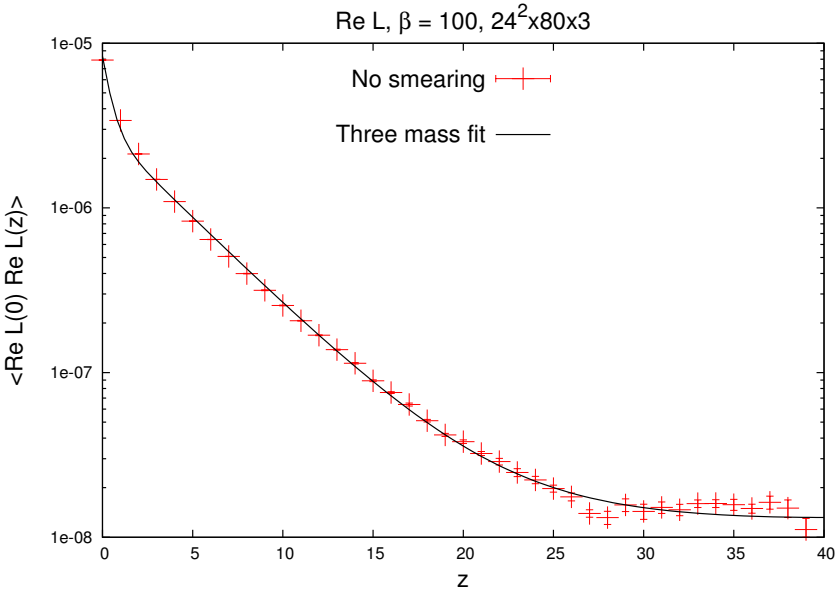
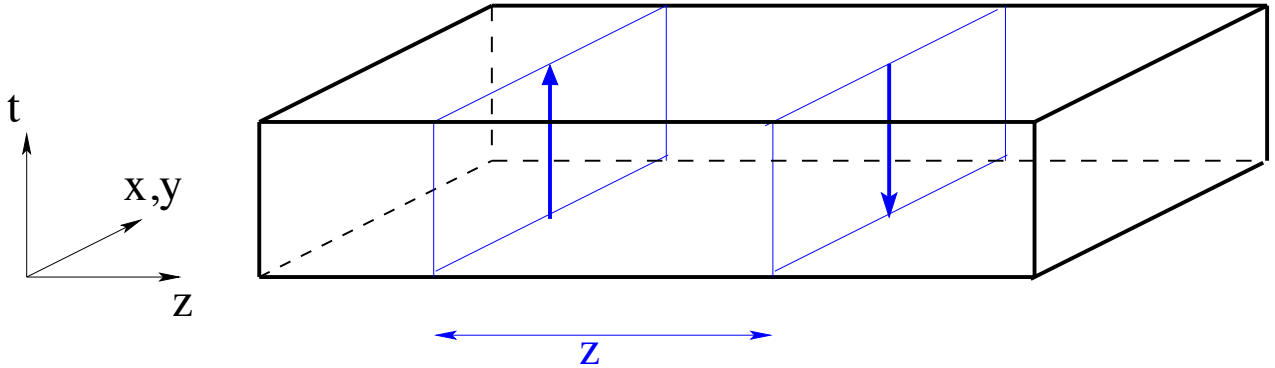




Symmetries

- Reversal of Euclidean time "R": $t \to -t,\ A_0 \to -A_0,\ \mathrm{Tr}L \to \mathrm{Tr}L^\dagger$
- MQCD ($3d$ effective theory, no A_0) is "R"-even $\to$ scale $g^2 T$ in "R"-even observables only



Define: $\mathrm{Re}L = \frac{L+L^\dagger}{2}$ R-even, $\mathrm{Im}L = \frac{L-L^\dagger}{2i}$ R-odd

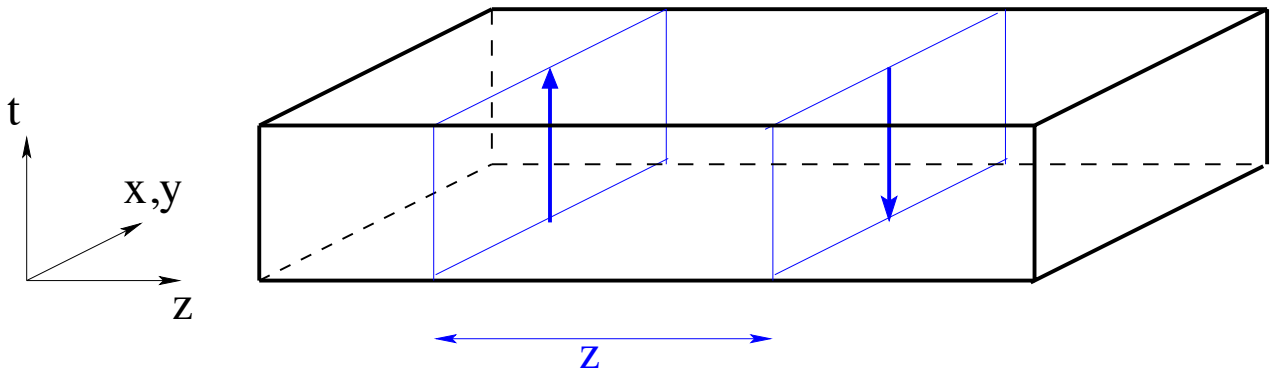
$\left\langle \left[\sum_{xy} \mathrm{TrRe}L(x,y,0)\right] \left[\sum_{xy} \mathrm{TrRe}L(x,y,0)\right] \right\rangle_c$ projects on $\{2\pi T, gT, g^2 T\}$

$\left\langle \left[\sum_{xy} \mathrm{TrIm}L(x,y,0)\right] \left[\sum_{xy} \mathrm{TrIm}L(x,y,0)\right] \right\rangle$ projects on $\{2\pi T, gT\}$

..., Arnold and Yaffe, ...

Expectations

- Polyakov loop: $L = \exp\left(i\bar{A}_0\right) \underset{\bar{A}_0 \ll 1}{\approx} 1 + i\bar{A}_0 - \frac{\bar{A}_0^2}{2} - \frac{i\bar{A}_0^3}{6} + \cdots$ with $\mathrm{Tr}\bar{A}_0 = 0$
- At $\mathcal{O}(gT)$: $\mathrm{Tr}\mathrm{Re}L \sim \bar{A}_0^2 \to$ mass $2m_E$
  $\mathrm{Tr}\mathrm{Im}L \sim \bar{A}_0^3 \to$ mass $3m_E$

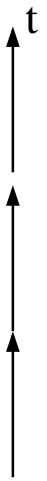


$$\left\langle \left[\sum_{xy} \mathrm{Tr}\mathrm{Re}L(x,y,0)\right] \left[\sum_{xy} \mathrm{Tr}\mathrm{Re}L(x,y,0)\right] \right\rangle_c \quad m_{\mathrm{eff}} \in \{\sim 2\pi T, 2m_E + \mathrm{corr.}, m_G(0^+)\}$$

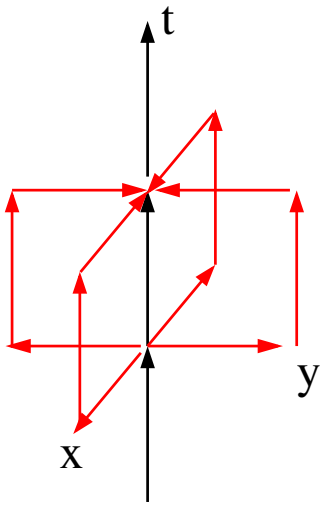
$$\left\langle \left[\sum_{xy} \mathrm{Tr}\mathrm{Im}L(x,y,0)\right] \left[\sum_{xy} \mathrm{Tr}\mathrm{Im}L(x,y,0)\right] \right\rangle \quad m_{\mathrm{eff}} \in \{\sim 2\pi T, 3m_E + \mathrm{corr.}\}$$

Subtlety

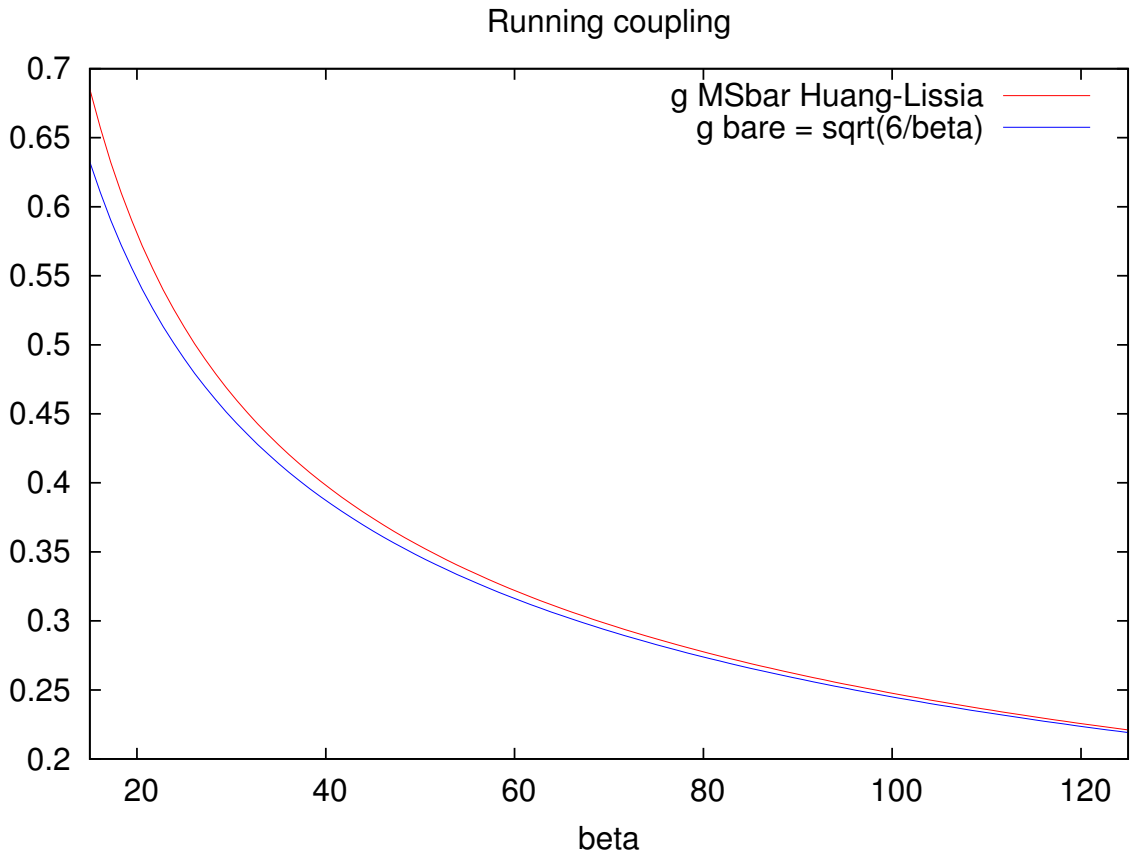
- Without smearing, Tr$\mathrm{Im}L$ is “R"-odd (changes sign under $t \to -t$)



- No longer true after smearing: changes sign under $(t \to -t, x \to -x, y \to -y)$ together
  Tr$\mathrm{Im}L$ is neither “R"-odd nor “R"-even
  Projects also onto lightest state (“R"-even glueball)

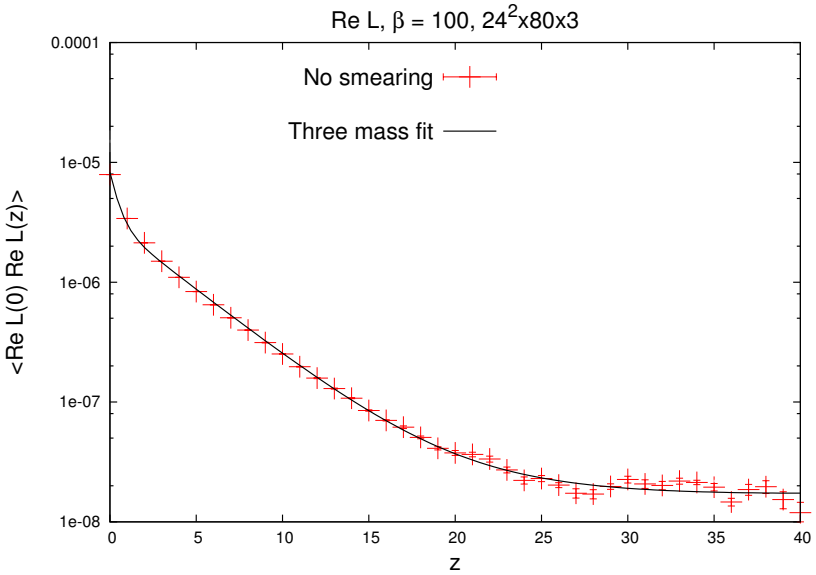


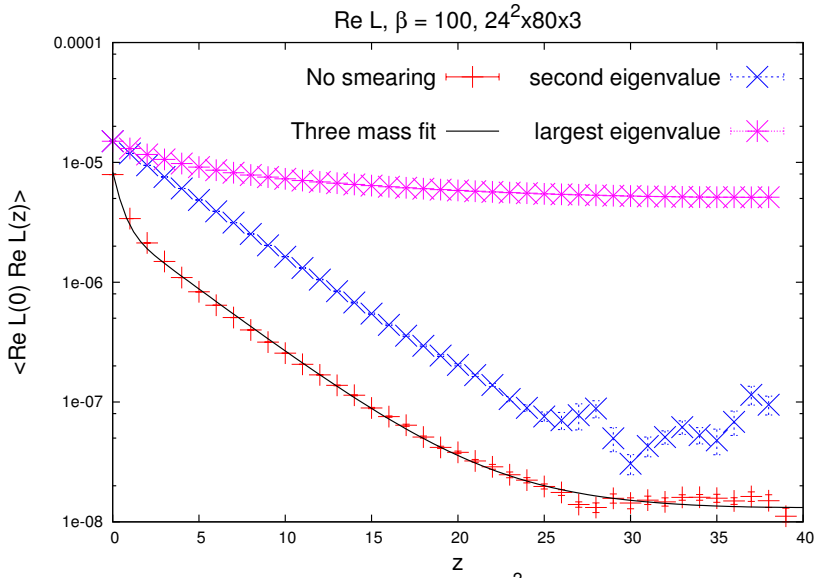
Results: $SU(3), 24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20, 120]$



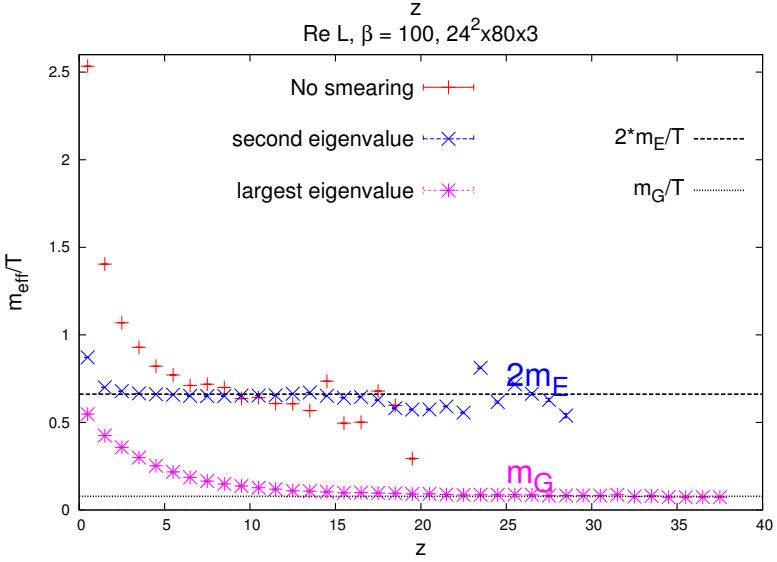
g chosen suitably small $(0.2 - 0.6)$ for a clear scale hierarchy
But $T \sim 10^9 - 10^{69}\, T_c$!!!

Results: $SU(3)$, $24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20, 120]$





- $\mathrm{Re}L \to m_{\mathrm{eff}} \in$
  $\{\sim 2\pi T, 2m_E + \mathrm{corr.}, m_G(0^+)\}$



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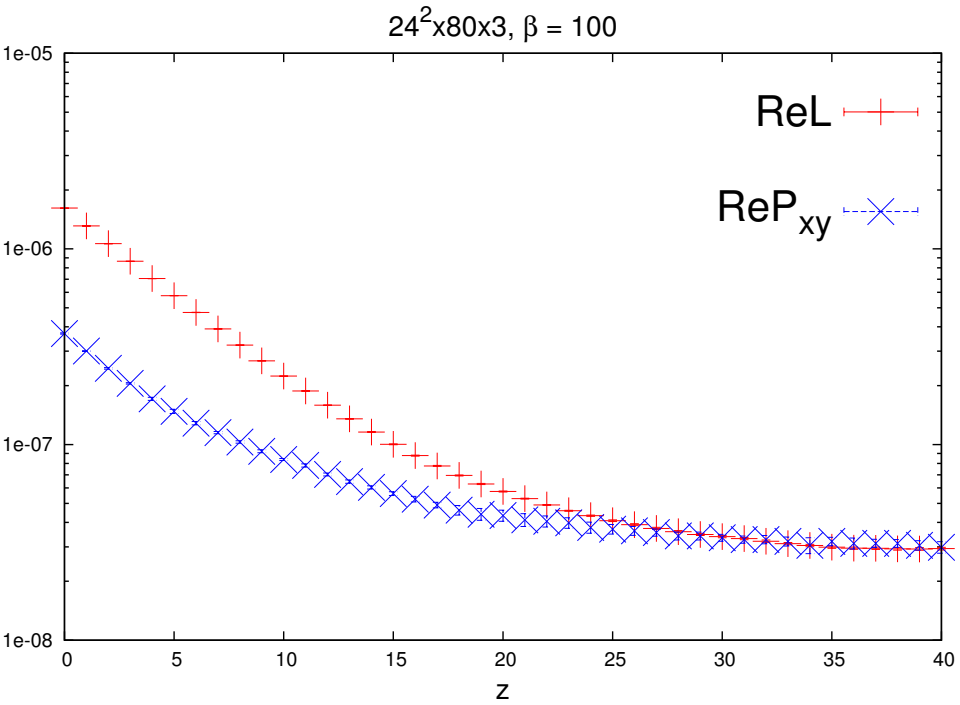
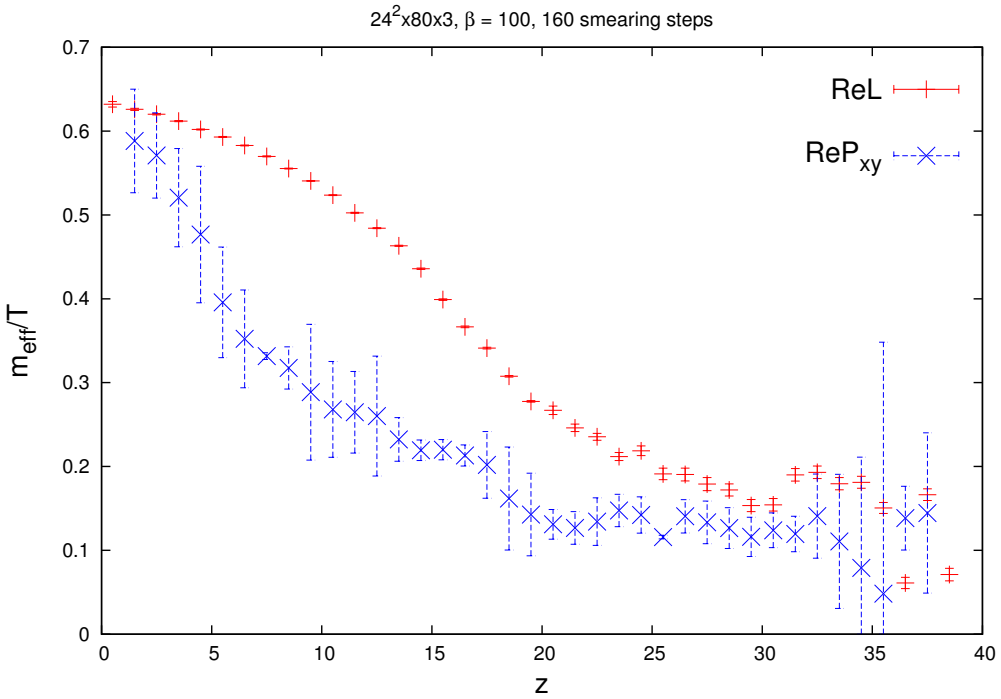
- Re$L \to m_{\text{eff}} \in \{\sim 2\pi T, 2m_E + \text{corr.}, m_G(0^+)\}$
- Crosscheck $m_G\,(0^+)$ by measuring correlator of ReTr Plaq$_{xy} \sim F_{xy}^2$





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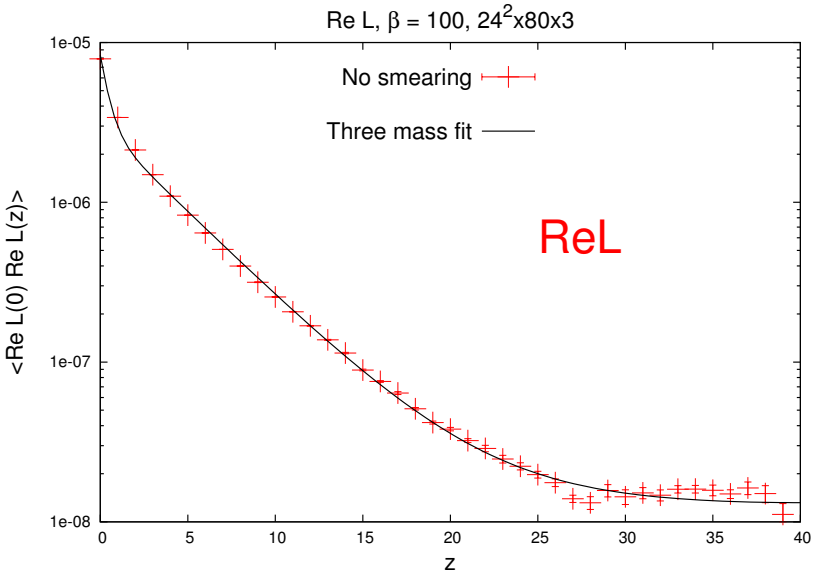
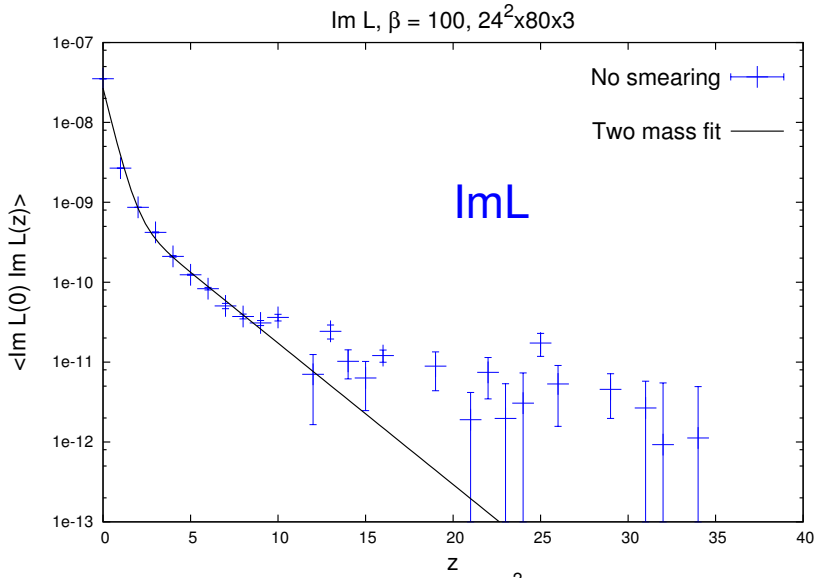
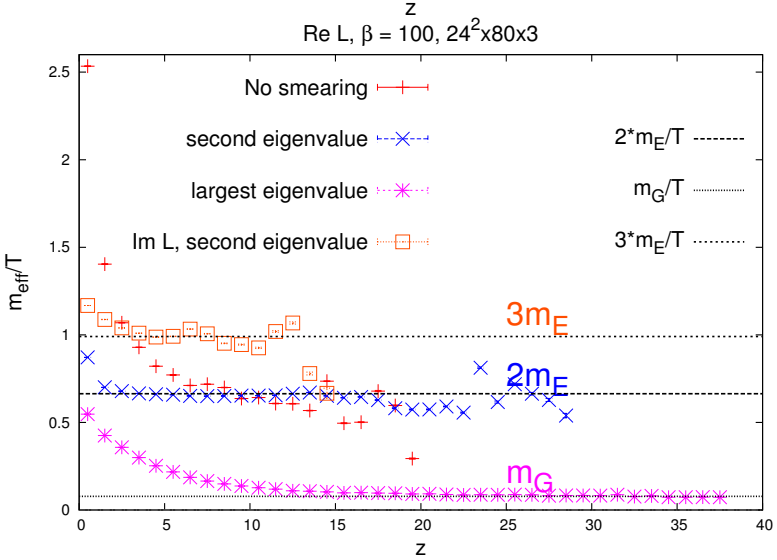


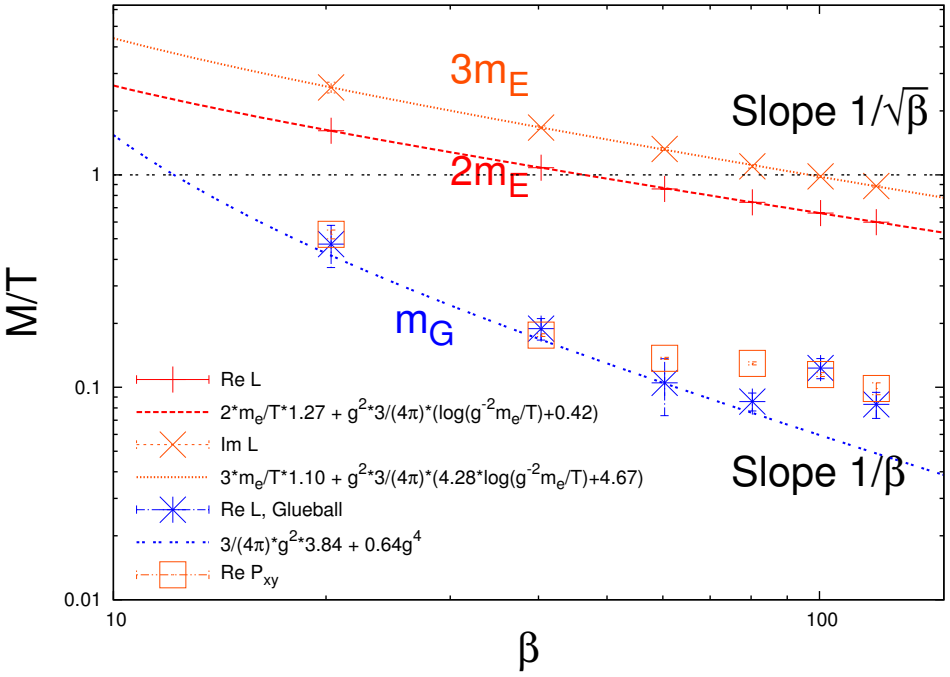


- $\mathrm{Im}L \to m_{\mathrm{eff}} \in$
  $\{\sim 2\pi T, 3m_E + \text{corr.}\}$
  (and m_G when smeared)



Results: $SU(3)$, $24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20, 120]$

- Masses versus β:
  - check $m_E \sim gT$, $m_G \sim g^2 T$
  - fit non-perturbative corrections to m_E
  - finite-size effects on m_G
  - + discretization errors $\mathcal{O}(1/\beta)$

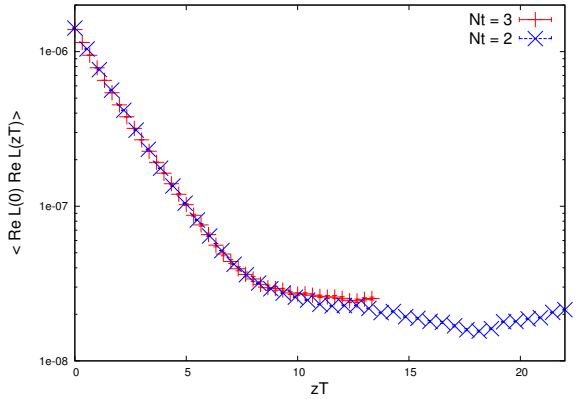


$$3m_E \quad \rightarrow \quad 3m_E \times 1.09(14) + \frac{3g^2}{4\pi}\left[\log\frac{m_E}{g^2 T} \times 5(5) + 4.05(80)\right]$$

$$2m_E \quad \rightarrow \quad 2m_E \times 1.27(1) + \frac{3g^2}{4\pi}\left[\log\frac{m_E}{g^2 T} + 0.33(12)\right]$$

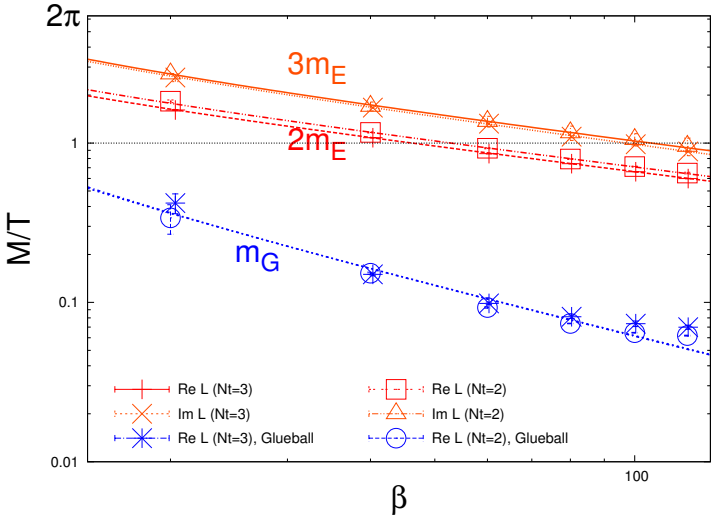
Results: $SU(3)$, $24^2 \times 80 \times 2$ and 3 lattices, $\beta = \frac{6}{g^2} \in [20, 120]$

- Continuum limit: compare $(N_t = 2, \beta)$ and $(N_t = 3, \beta + \Delta\beta)$



Correlators match after rescaling coordinates!

Small spectrum corrections, consistent with $1/N_t^2$

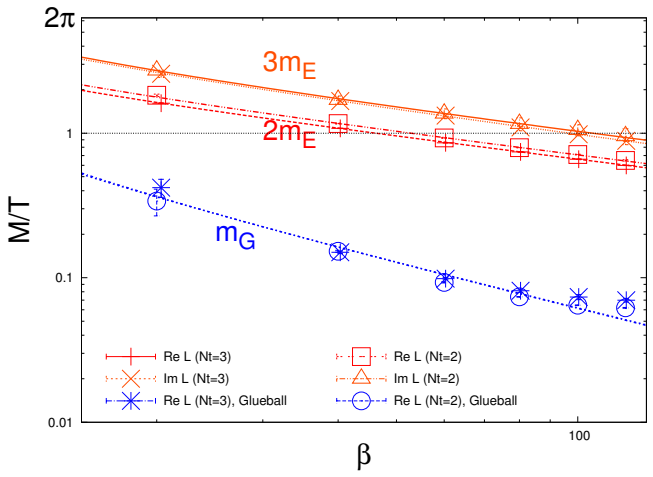


$$3m_E \to 3m_{E,\text{cont}} \times \{1.31(4)[N_t = 2], 1.09(14)[N_t = 3]\}$$

$$2m_E \to 2m_{E,\text{cont}} \times \{1.34(3)[N_t = 2], 1.27(1)[N_t = 3]\}$$

Summary

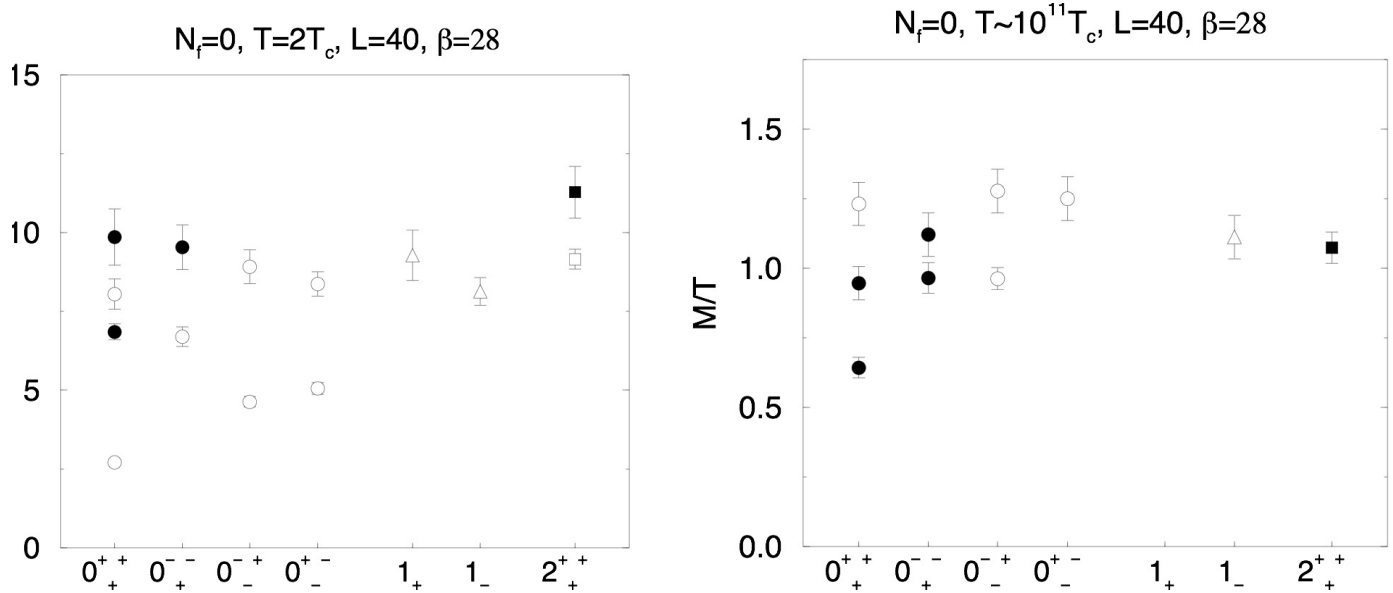
- ImL: $\frac{M(\mathrm{Tr}[A_0^3])}{T} = 3\frac{m_E}{T} + \frac{g^2 N_c}{4\pi}\left(b_3 \log\frac{m_E}{g^2 T} + c_3\right)$, b_3, c_3 non-perturbative
- ReL: $\frac{M(\mathrm{Tr}[A_0^2])}{T} = 2\frac{m_E}{T} + \frac{g^2 N_c}{4\pi}\left(\log\frac{m_E}{g^2 T} + c_2\right)$, c_2 non-perturbative and $m_G(0^+)$
- $\frac{M}{T} \gtrsim 1$: No clear scale hierarchy even at $T \sim 10^{30}\, T_c$



Lesson: success of effective $3d$ description at $T \gtrsim 3 - 10\, T_c$ does not necessarily imply scale hierarchy

Compare with 3d simulations of EQCD:

hep-ph/0004060 (Hart et al)

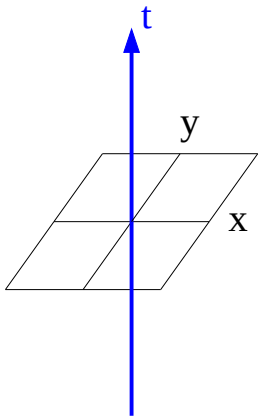


The spectrum of screening masses in various quantum number channels at $N_f = 0$, $T = 2T_c$ (left), $N_f = 0$, $T \sim 10^{11} T_c$ (right). Filled symbols denote $3d$ glueball states, which have become the lightest excitations at $T \sim 10^{11} T_c$

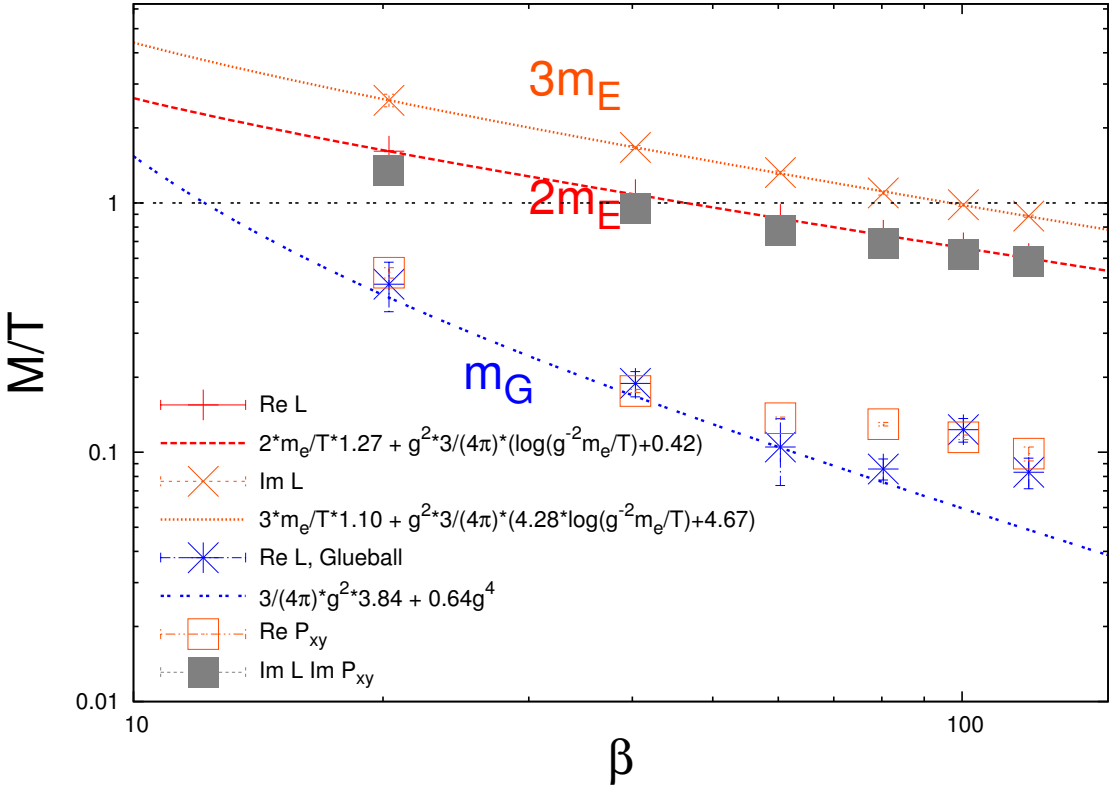
Hierarchy $2\pi T \gg m_E \gg m_G$ more or less OK at $T \sim 10^{11} T_c$
Inverted into $m_E \sim 2\pi T \lesssim m_G$ at $T = 2T_c$

Supplement with center degrees of freedom: 0801.1566 (Kurkela et al)

Mystery?



- Measure mass from $\mathrm{Im}L\,\mathrm{ImPlaq}_{xy} \sim \mathrm{ReTr}\,[A_0 F_{xy}] \sim m_E$
- Expect $\frac{M(\mathrm{Tr}[A_0 F_{xy}])}{T} = \frac{m_E}{T} + \frac{g^2 N_c}{4\pi}\left(\log\frac{m_E}{g^2 T} + c_1\right)$



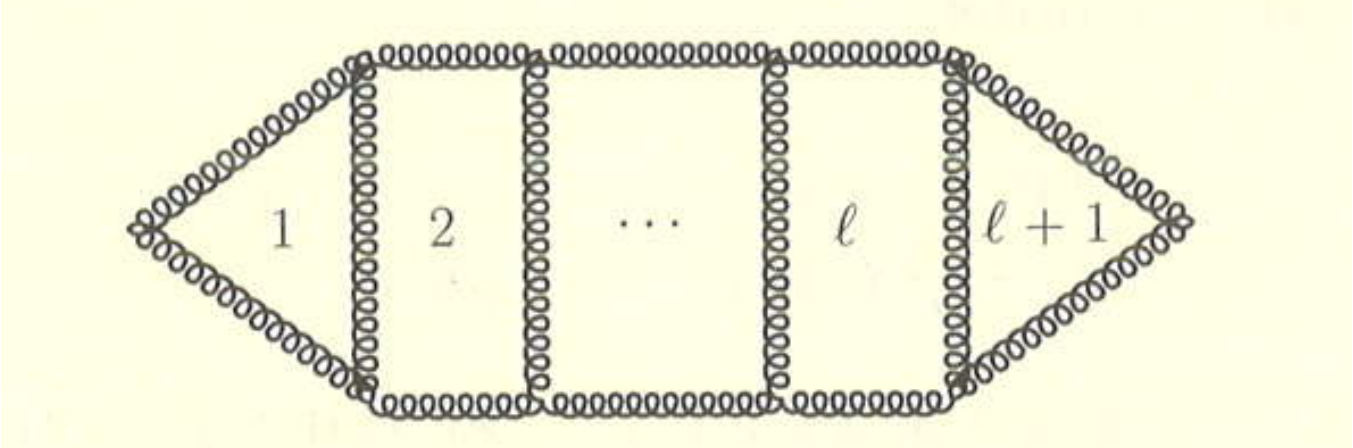
Too heavy?

"m_E" approaches "$2m_E$"

as $\beta \to \infty$??

Non-perturbative $\mathcal{O}(g^2 T)$: Linde problem

- Consider perturbative expansion of free energy (pressure)



$(l+1)$ loops, $2l$ vertices, $3l$ propagators: $g^{2l}\left(T\int \mathrm{d}^3k\right)^{l+1} k^{2l}\left(k^2+m^2\right)^{-3l}$

$m=0 \to \int \mathrm{d}k\, k^{3l+2-4l}$ IR-divergent if $l \geq 3$ ie. non-perturbative at $\mathcal{O}(g^6)$

Divergence cured by non-perturbative mass $m_G \sim \mathcal{O}(g^2 T)$ mass gap of $3d$ theory ($3d$ glueball)