Investigation of the axial anomaly in high temperature QCD on the lattice

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Introduction

- Renormalization Group studies[1,2] on models with the same symmetries as QCD suggest that the order of the chiral phase transition for $N_f = 2$ QCD at zero baryon density depends on the magnitude of the axial anomaly, $U_A(1)$.
- If $U_A(1)$ is not restored at the chiral phase transition ⇒ second order transition.
- Existence of critical point expected.
- For $2+1$ flavour QCD, the light quark masses $m_l << \Lambda_{QCD}$ ⇒ chiral symmetry for the light quark sector still relevant.
- We investigate in this work, the role of the $U_A(1)$ for physical quark masses for $N_f = 2+1$ QCD using non-perturbative lattice techniques, near and above the chiral cross-over temperature T_c.
- In particular, it would give us an insight whether the critical end-point exists.

The Set-up

- Highly Improved Staggered Quark(HISQ) discretization is used quite extensively for QCD thermodynamics.
- Has least taste symmetry violations on the lattice.
- Continuum extrapolated results for T_c, χ_{2B} are known ⇒ in excellent agreement with other improved staggered operators like ASQTAD and stout smeared.
- We use the overlap fermion operator[3] to study the underlying topology of the HISQ configurations by looking at its eigenvalue distribution.

Configurations used

- We used the $32^3 \times 8$ HISQ configurations generated by the Bielefeld-BNL collaboration.
- Volume: $m_\pi L > 3$.
- $N_f = 2+1$: strange quark mass is at physical value, $m_s/m_l = 20 \rightarrow$ pion mass $= 160$ MeV.

Implementing the Overlap operator

$$D_{ov} = M\left[1 + \gamma_5 sgn(\gamma_5 D_W)\right].$$

- Lowest 20 eigenvalues of $\gamma_5 D_W$ computed with $\epsilon^2 < 10^{-16}$.
- For these lowest modes sign function was computed explicitly.
- For the higher modes, sign function approximated as a Zolotarev Rational Polynomial with 15 terms.
- The sign function is computed as precise as 10^{-10}.

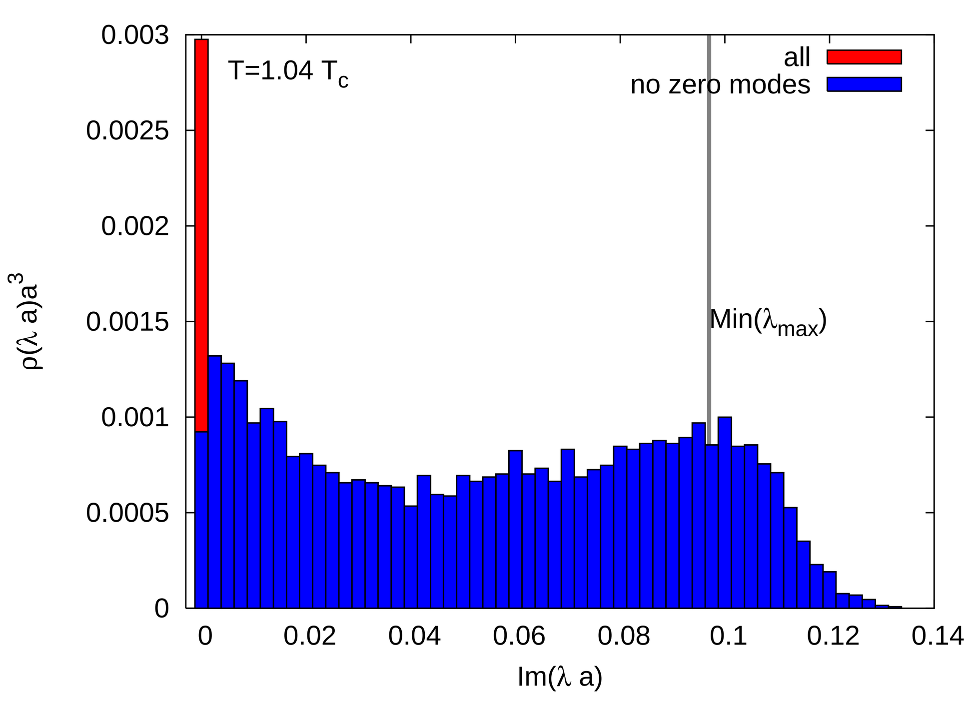
Eigenvalues of the overlap operator on HISQ sea

Computing Eigenvalues

- The eigenvalues computed using Ritz-minimization with Kalkreuter-Simma algorithm[4].
- Convergence criterion : $\epsilon^2 < 10^{-8}$.
- Eigenvalue statistics

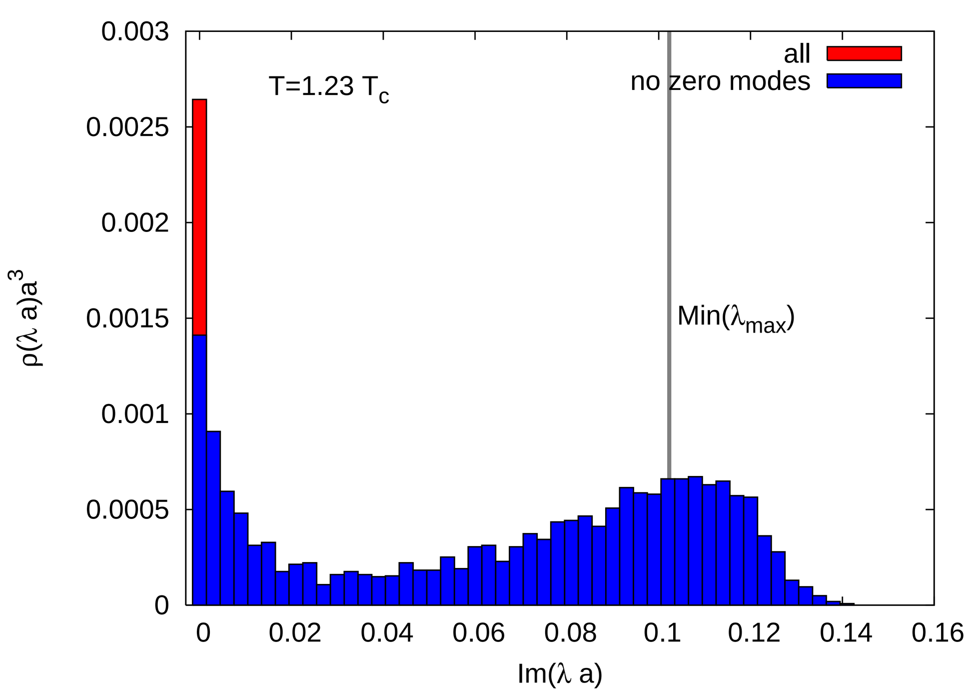
T	#configs	# eigenvalues/config
1.04 T_c	100	100
1.23 T_c	100	50
1.50 T_c	100	50

- The computations were done on the GPU cluster at the Bielefeld University.



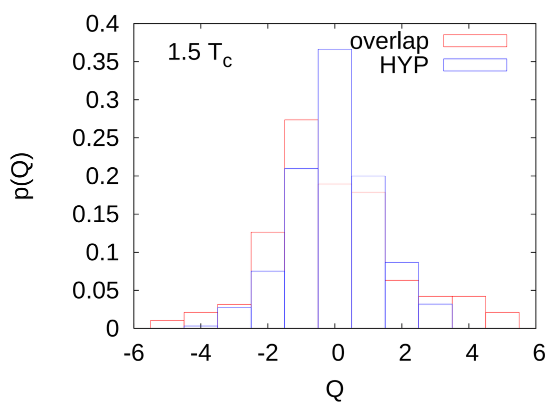
Why the overlap operator?

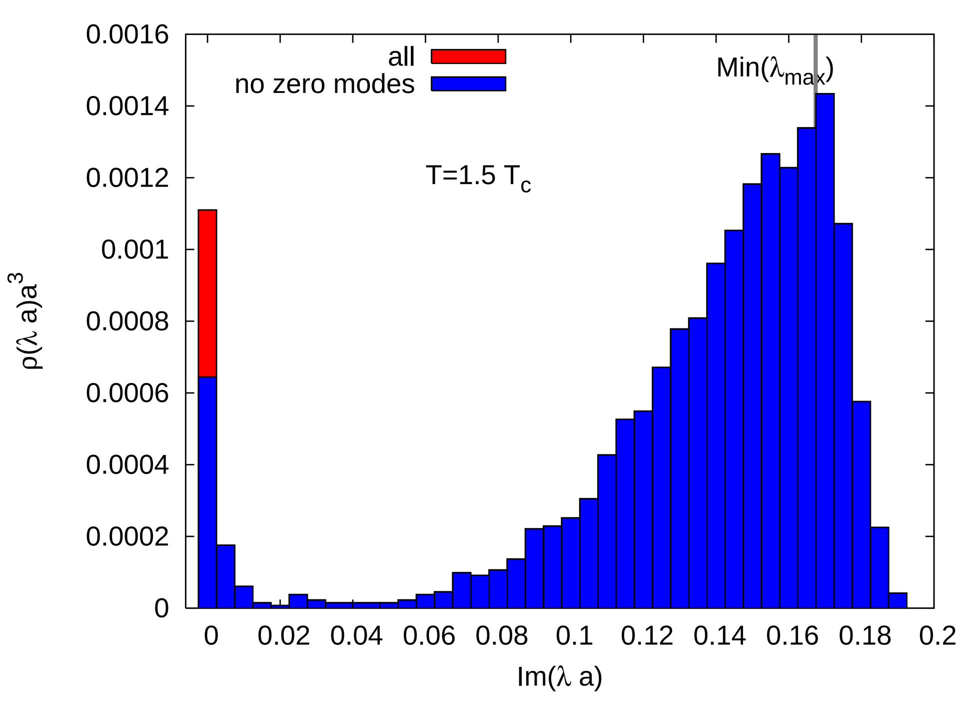
- Overlap operator satifies an index theorem on the lattice ⇒ zero modes of the overlap operator related to the non-trivial topology of the gauge fields[5].
- Our idea to use overlap valence quarks is to get a clear separation between the zero and near-zero modes.



Our observations

- Significant fraction of the configurations have true zero-modes.
- Cross-checked by comparing the topological charge measured from the $F\tilde{F}$ using HYP smearing on the same configurations[6].



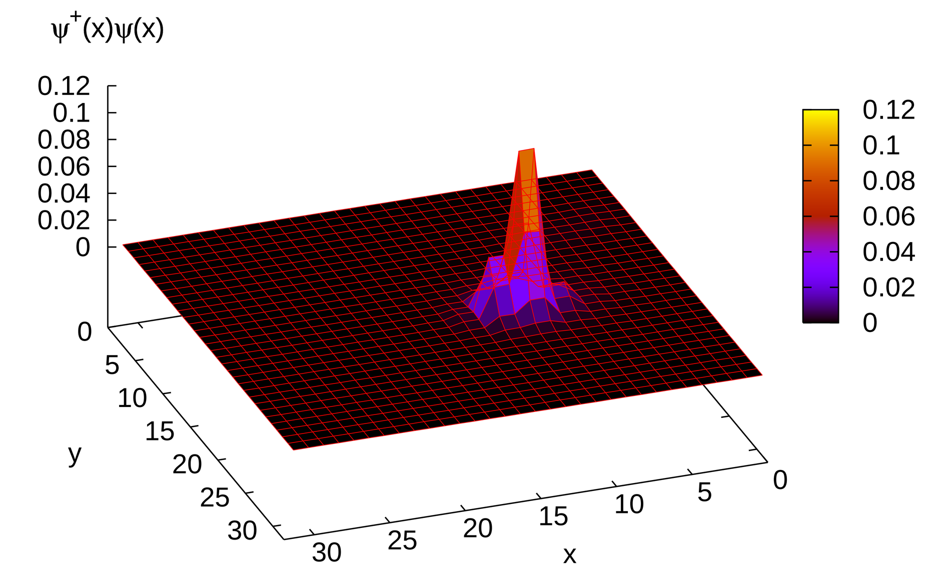


Important

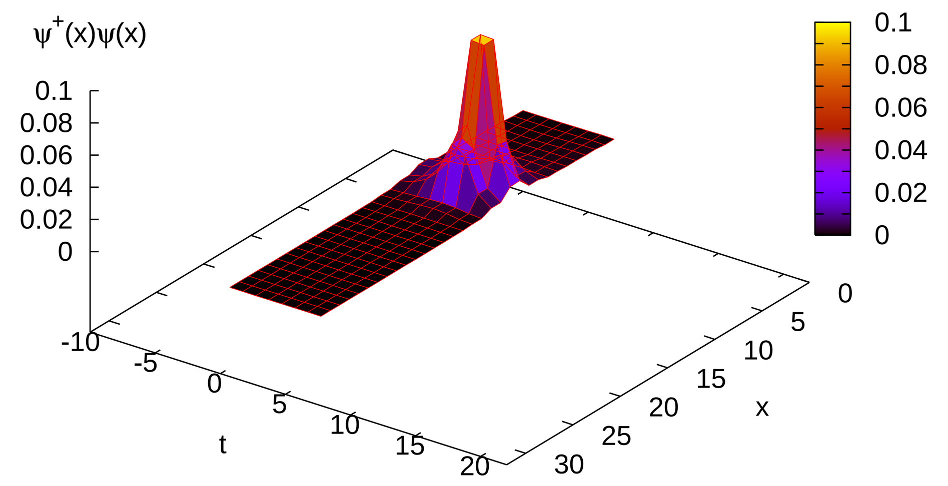
- Clear presence of a finite density of near-zero modes even at 1.5 T_c.
- No signal of a gap opening up ⇒ $U_A(1)$ is not restored.

Profile of the zero modes at 1.5 T_c

The zero modes are localized in space with a well defined peak.



These are localized in the temporal direction as well.



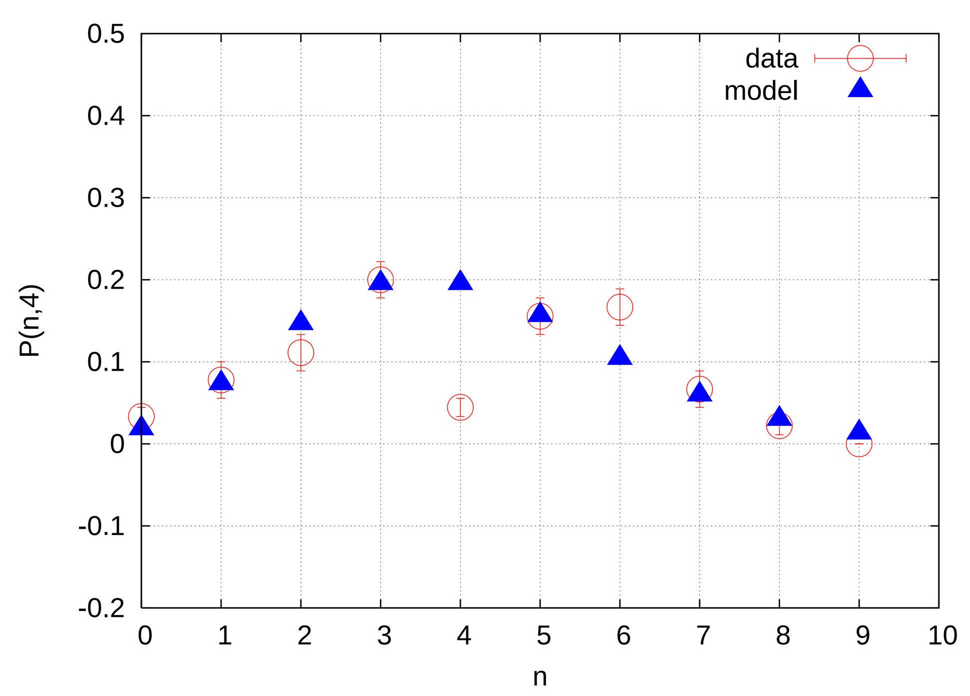
Radii of these profile is smaller than box size and of the order of 1/3 fm
⇒ More in agreement with dilute instanton gas model.

A closer look at the near-zero modes at 1.5 T_c

- We compare the presence of near-zero modes with the expectation from dilute instanton gas model.
- If n = number of instantons+anti-instantons interacting weakly ⇒ they should follow Poisson distribution,

$$P(n, \langle n \rangle) = \langle n \rangle^n \mathrm{e}^{-\langle n \rangle}/\mathrm{n}!$$

- This would result in $\langle n \rangle = \langle n^2 \rangle$.
- At 1.5 T_c, for Im$\lambda a < 0.036$, indeed $\langle n \rangle = 4 = \langle n^2 \rangle$.



- Such near-zero mode peak observed in the eigenvalue spectrum of $2+1$ flavour dynamical domain wall fermions above T_c, as well[7].

Summary

- The 2+1 flavour HISQ configurations on a large lattice volume used extensively for QCD thermodynamics, show a significant presence of zero modes even beyond T_c.
- The fermion zero modes are localized both in the spatial and 'temporal' directions.
- Even more important are the presence of near-zero modes at 1.5 T_c.
- We do not observe a gap in the low-lying eigenvalue spectrum even at 1.5 T_c ⇒ $U_A(1)$ is not restored.
- The presence of near-zero modes are consistent with the expectation from the dilute instanton gas model.

References

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