Introduction

- Renormalization Group studies\[1,2\] on models with the same symmetries as QCD suggest that the order of the chiral phase transition for $N_f = 2$ QCD at zero baryon density depends on the magnitude of the axial anomaly, $U_1(1)$.
- If $U_1(1)$ is not restored at the chiral phase transition $\Rightarrow$ second order transition.
- Existence of critical point expected.
- For $2 + 1$ flavour QCD, the light quark masses $m_i < < \Lambda_{QCD} = \text{chiral symmetry for the light quark sector still relevant.}$
- We investigate in this work, the role of the $U_1(1)$ for physical quark masses for $N_f = 2 + 1$ QCD using non-perturbative lattice techniques, near and above the chiral cross-over temperature $T_c$.
- In particular, it would give us an insight whether the critical end-point exists.

The Set-up

- Highly Improved Staggered Quark(HISQ) discretization is used quite extensively for QCD thermodynamics.
- Has least taste symmetry violations on the lattice.
- Continuum extrapolated results for $T_c, \chi_{SF}$ are known $\Rightarrow$ excellent agreement with other improved staggered operators like asqtad and stout smeared.
- We use the overlap fermion operator\[3\] to study the underlying topology of the HISQ configurations by looking at its eigenvalue distribution.

Configurations used

- We used the $32^3 	imes 8$ HISQ configurations generated by the Bielefeld-BNL collaboration.
- $N_f = 2 + 1$: strange quark mass is at physical value, $m_s/m_u = 20 \rightarrow$ pion mass $= 160$ MeV.

Implementing the Overlap operator

\[ D_{\text{ov}} = M \left[ 1 + \gamma_5 \text{sgn}(\gamma_5 D_{\text{W}}) \right] \]

- Lowest 20 eigenvalues of $\gamma_5 D_{\text{W}}$ computed with $\gamma^2 < 10^{-10}$.
- For these lowest modes sign function was computed explicitly.
- For the higher modes, sign function approximated as a Zolotarev Rational Polynomial with 15 terms.
- The sign function is computed as precise as $10^{-10}$.

Eigenvalues of the overlap operator on HISQ sea

Computing Eigenvalues

- The eigenvalues computed using Ritz minimization with Kalkreuter-Simma algorithm\[4\].
- Convergence criterion: $\gamma^2 < 10^{-4}$.
- Eigenvalue statistics

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<thead>
<tr>
<th>$T_c$</th>
<th>configs</th>
<th>eigenvalues/config</th>
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<tbody>
<tr>
<td>$1.04 T_c$</td>
<td>100</td>
<td>100</td>
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<tr>
<td>$1.23 T_c$</td>
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- The computations were done on the GPU cluster at the Bielefeld University.

Why the overlap operator?

- Overlap operator satisfies an index theorem on the lattice $\Rightarrow$ zero modes of the overlap operator related to the non-trivial topology of the gauge fields\[5\].
- Our idea to use overlap valence quarks is to get a clear separation between the zero and near-zero modes.

Our observations

- Significant fraction of the configurations have true zero-modes.
- Cross-checked by comparing the topological charge measured from the $FF$ using HYP smearing on the same configurations\[6\].

Important

- Clear presence of a finite density of near-zero modes even at $1.5 T_c$.
- No signal of a gap opening up $\Rightarrow U_1(1)$ is not restored.

Profile of the zero modes at $1.5 T_c$

- The zero modes are localized in space with a well defined peak.
- These are localized in the temporal direction as well.

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A closer look at the near-zero modes at $1.5 T_c$

- We compare the presence of near-zero modes with the expectation from dilute instanton gas model.

\[ P(n, \langle n \rangle) = \langle n \rangle^{-5/2} e^{-\langle n \rangle}/\pi! \]

- This would result in $\langle n \rangle = (\pi^2)^{1/5}/\pi!$
- At $1.5 T_c$, for $\text{Im} \lambda A < 0.036$, indeed $\langle n \rangle = 4 = (\pi^2)^{1/5}/\pi!$

- Such near-zero mode peak observed in the eigenvalue spectrum of $2 + 1$ flavour dynamical domain wall fermions above $T_c$, as well\[7\].

Summary

- The $2+1$ flavour HISQ configurations on a large lattice volume used extensively for QCD thermodynamics, show a significant presence of zero modes even beyond $T_c$.
- The fermion zero modes are localized both in the spatial and ‘temporal’ directions.
- Even more important are the presence of near-zero modes at $1.5 T_c$.
- We do not observe a gap in the low-lying eigenvalue spectrum even at $1.5 T_c \Rightarrow U_1(1)$ is not restored.
- The presence of near-zero modes are consistent with the expectation from the dilute instanton gas model.

References