Simulating Abelian Gauge-Higgs models using the worm algorithm

Y. Delgado, C. Gattringer, A. Schmidt
Karl-Franzens-Universität Graz

Basic idea

- Lattice QCD is a reliable non-perturbative tool in hadron physics.
- At finite density Lattice QCD faces the complex phase problem
  $\Rightarrow$ new ideas!
- Complex phase problem is basis dependent
  $\Rightarrow$ new degrees of freedom: **dual representation**
- Simulation: local Metropolis and **worm algorithm** [1, 2].

U(1) gauge-Higgs model with 2 flavors

The theory is described by the action:

$$S_G = -\frac{\beta}{2}\sum_x \sum_{\nu<\rho}\left[U_{x,\nu\rho} + U^*_{x,\nu\rho}\right]$$

$$S_H^f = M_f \sum_x |\phi_x^f|^2 - \sum_{x,\nu}\left[e^{-\mu_f\delta_{\nu 4}}\phi_x^{f*}U_{x,\nu}\phi^f_{x+\hat\nu} + e^{\mu_f\delta_{\nu 4}}\phi_x^{f*}U^*_{x-\hat\nu,\nu}\phi^f_{x+\hat\nu}\right]$$

$$\phi_x^f \in \mathbb{C}; \quad U_{x,\nu} = e^{iA_\nu} \in U(1),\ A_\nu \in [-\pi,\pi]$$

Dual representation

General idea of the dual representation [3]:

- A single nearest neighbor term:

$$e^{e^{-\mu\delta_{\nu 4}}\phi_x^* U_{x,\nu}\phi_{x+\hat\nu}} = \sum_{l_{x,\nu}} \frac{(e^{-\mu\delta_{\nu 4}})^{l_{x,\nu}}}{l_{x,\nu}!}(U_{x,\nu})^{l_{x,\nu}}(\phi_x^*)^{l_{x,\nu}}(\phi_{x+\hat\nu})^{l_{x,\nu}}$$

- A single plaquette term:

$$e^{\beta U_{x,\nu}U_{x+\hat\nu,\rho}U^*_{x+\hat\rho,\nu}U^*_{x,\rho}} = \sum_{p_{x,\nu\rho}} \frac{\beta^{p_{x,\nu\rho}}}{p_{x,\nu\rho}!}\left[U_{x,\nu}U_{x+\hat\nu,\rho}U^*_{x+\hat\rho,\nu}U^*_{x,\rho}\right]^{p_{x,\nu\rho}}$$

- Integrate out fields $\rightarrow$ new degrees of freedom: $l^f_{x,\nu},\ p_{x\nu\rho} \in \mathbb{Z}$
- New partition sum:

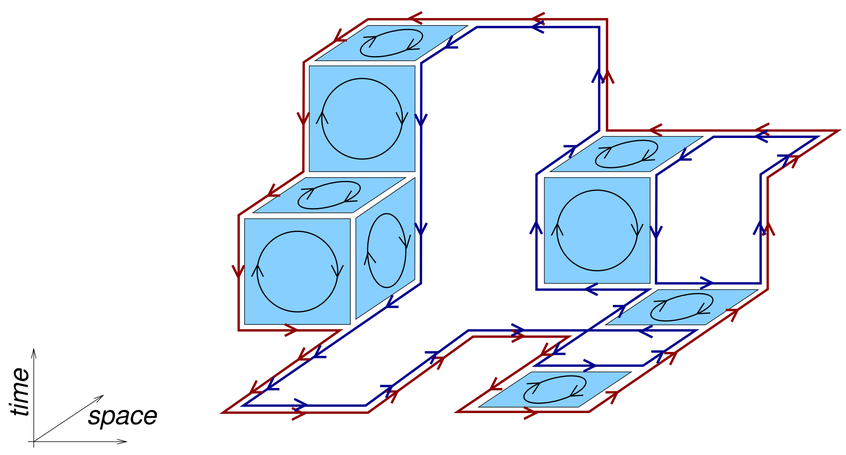
$$Z \propto \sum_{\{p,l^1,l^2\}} \mathcal{W}[p,l^1,l^2]\ \mathcal{C}_S[l^1]\ \mathcal{C}_S[l^2]\ \mathcal{C}_L[p,l^1,l^2]$$

  - $\mathcal{W}[p,l^1,l^2]$: positive weight factor (**sign problem solved**).
  - $\mathcal{C}_S[l^f]$: site constraint $\rightarrow$ matter loops.

$$\forall x:\ \delta\left(\sum_\nu [l_{x,\nu} - l_{x-\hat\nu,\nu}]\right)$$

  - $\mathcal{C}_L[p,l^1,l^2]$: link constraint $\rightarrow$ gauge surfaces.

$$\forall x,\nu:\ \delta\left(\sum_{\rho:\nu<\rho}[p_{x,\nu\rho} - p_{x-\hat\rho,\nu\rho}] - \sum_{\rho:\nu>\rho}[p_{x,\rho\nu} - p_{x-\hat\rho,\rho\nu}] + l^1_{x,\nu} - l^2_{x,\nu}\right)$$

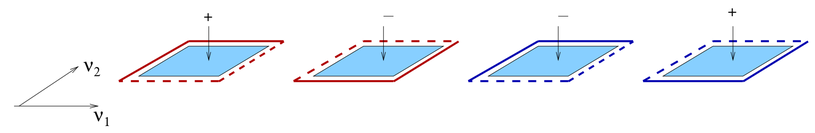


References

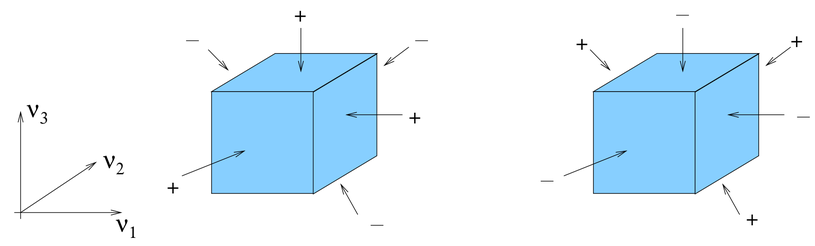
[1] N. Prokof'ev, B. Svistunov, Phys. Rev. Lett. **87** (2001) 160601.
[2] Y. D. Mercado, C. Gattringer and A. Schmidt, Comput. Phys. Commun. **184** (2013) 1535
[3] A. Patel, Nucl. Phys.B **243** (1984) 411; Phys. Lett. B **139**(1984) 394. T. DeGrand, C. DeTar, Nucl. Phys. B **225**(1983) 590. C. Gattringer and A. Schmidt, Phys. Rev. D **86** (2012) 094506

Local Metropolis update (LMA)

Plaquette update:



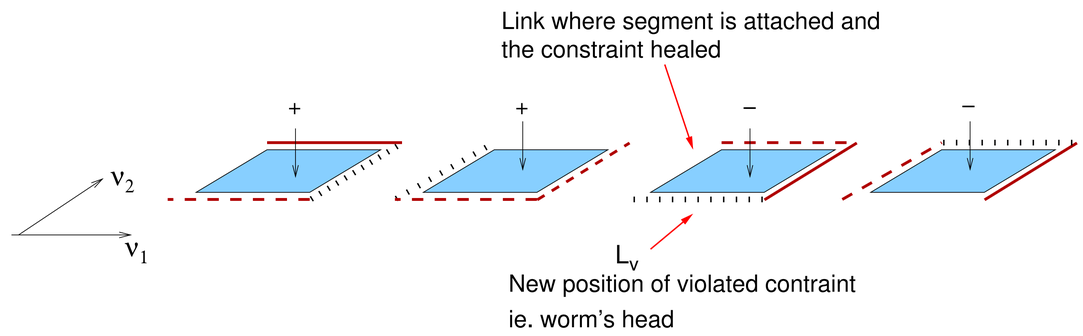
Cube update:



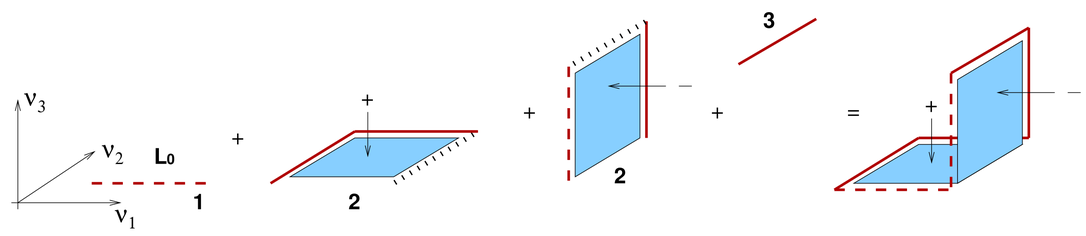
Winding loops

Surface worm algorithm (SWA)

The SWA is constructed by breaking up the plaquette update (segments):

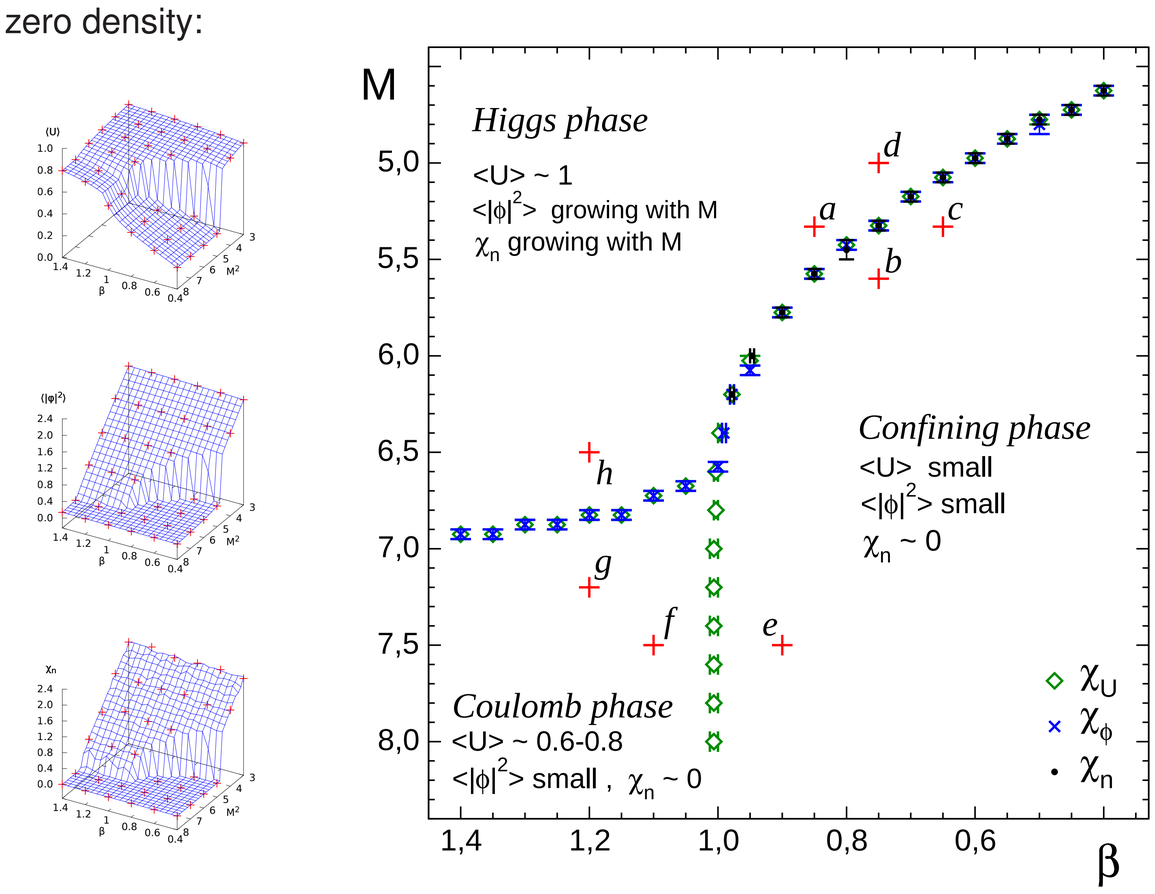


The worm algorithm we use produces admissible configurations using 3 different steps which we illustrate below. The worm starts inserting a link at a random position L_0 (1). It may insert a new segment at L_v, healing the constraints at this position and then move to one of other three links of the segment (positions 2) until it closes healing the constraint at L_v (3).



Results: $M_1 = M_2 = M$

At zero density:



At finite chemical potential: