Effects of baryon number density fluctuation around QCD critical point

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Based on
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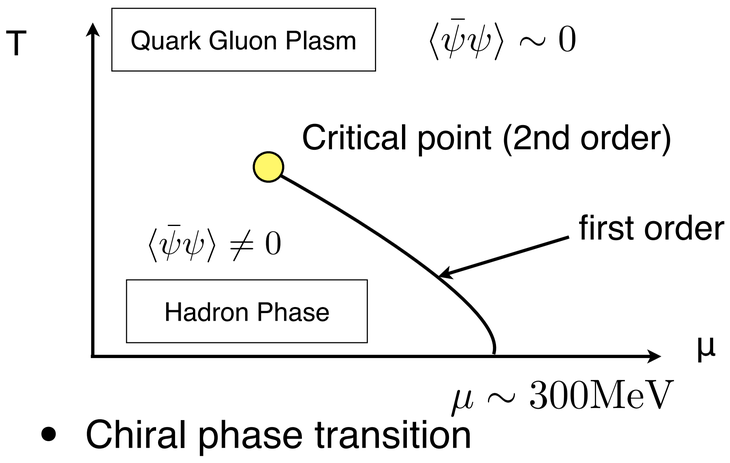
Motivation

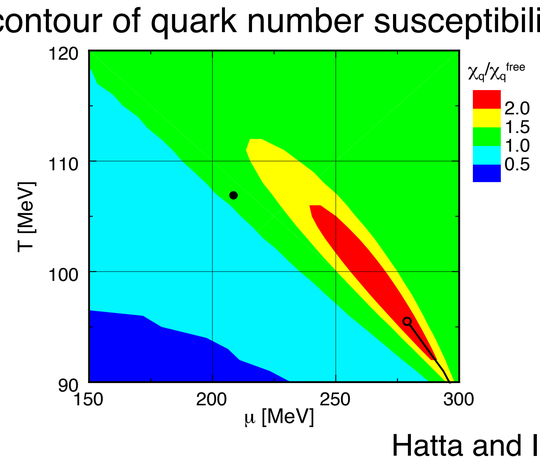
QCD phase diagram



- Chiral phase transition
- Order parameter $\langle\bar\psi\psi\rangle$
- 1st order boundary and critical point (2nd order)

Critical region

contour of quark number susceptibility



$$\chi_q = \frac{\partial^2 P}{\partial \mu^2}$$

Hatta and Ikeda (2003)

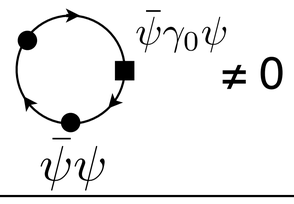
- Both chiral and quark-number susceptibilities diverge near the critical point.
  $\langle(\bar\psi\psi)^2\rangle,\ \langle(\bar\psi\gamma_0\psi)^2\rangle \gg 1$
- We need to evaluate the size or form of the critical region.

Landau potential at finite density

$$U = a\sigma^2 + b\sigma^4 + c\sigma^6 - d\sigma \quad (\mu = 0)$$

$\sigma \sim \bar\psi\psi$(chiral order parameter)

$\mu \neq 0$ and $m_q \neq 0$



$\mu \neq 0$

Fujii, Ohtani(2004)
Son, Staphanov(2004)

$$U = a\sigma^2 + b\sigma^4 + c\sigma^6 - d\sigma + \varphi\sigma^2$$

$\varphi \sim \bar\psi\gamma_0\psi$ (density fluctuation)

- At finite μ, coupling between σ and quark-density (phonon) mode is inevitable.

Formalism

Quark-meson model (ψ, σ and π) with density fluctuation (φ)

$$\mathcal{L} = \bar\psi[i\partial\!\!\!/ - g_s(\sigma + i\gamma_5\vec\tau\cdot\vec\pi) + g_d\varphi\gamma_0]\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec\pi)^2 + \frac{1}{2}(\partial_\mu\varphi)^2 - U(\sigma^2+\vec\pi^2) - \frac{m_\varphi}{2}\varphi^2 + c\sigma$$

$\varphi \sim \bar\psi\gamma_0\psi$

g_d determines the strength of the coupling between σ and φ.

Functional renormalization group equation

$$k\partial_k\Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left[\frac{k\partial_k R_{kB}}{R_{kB} + \Gamma_k^{(0,2)}[\varphi]}\right] - \mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF} + \Gamma_k^{(2,0)}[\varphi]}\right]$$

C. Wetterich (1993)

R is arbitrary cutoff function. Our choices are

$$R_{kB} = (k^2 - \vec p^{\,2})\theta[k^2 - \vec p^{\,2}],\ R_{kF} = ik\frac{\vec{p\!\!\!/}}{|\vec p|}\theta[k^2 - \vec p^{\,2}]$$

D. Litim (2000)

Scale dependent effective action

$\Gamma_{k=\Lambda}[\phi] = S[\phi]$ UV: classical

↓

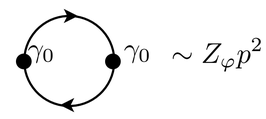
$\Gamma_{k=0}[\phi] = \Gamma[\phi]$ IR: quantum

Local potential approximation (LPA)

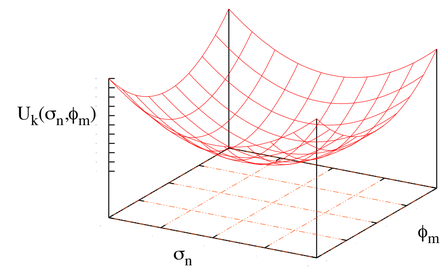
$$\Gamma_k^{LPA} = \frac{Z_{\sigma k}}{2}\partial\sigma\partial\sigma + \frac{Z_{\pi k}}{2}\partial\vec\pi\partial\vec\pi + \frac{Z_{\varphi k}}{2}\partial\varphi\partial\varphi - U_k(\sigma^2+\pi^2,\varphi) + c\sigma$$

$$Z_{\sigma k} = Z_{\pi k} = Z_{\varphi k} = 1$$

We solve functional differential equation for the scale dependent potential U_k in the 2D grid (σ_n,ϕ_m) space.



flows of wave function renormalization are neglected



Conditions for the order parameters: $\frac{\partial U_{k=0}}{\partial\sigma} = \frac{\partial U_{k=0}}{\partial\varphi} = 0$

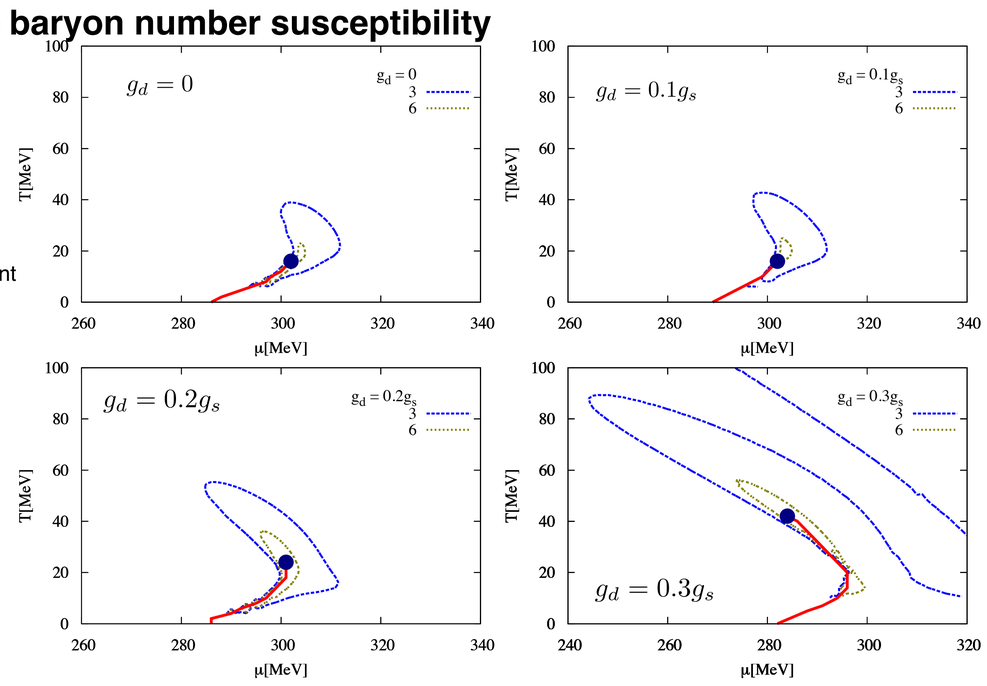
Results

Contour of the baryon number susceptibility

$$R_q = \chi_q/\chi_q^{\mathrm{free}}$$

1st order

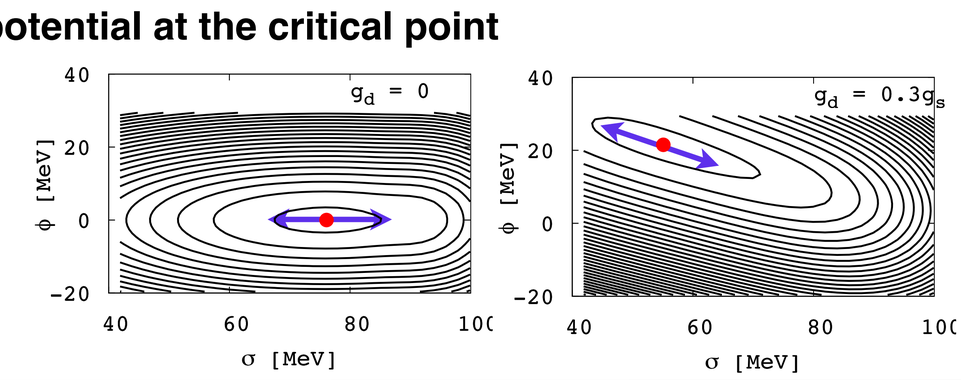
critical point



- The position of the CP moves gradually.
- The critical region is drastically expanded with g_d.

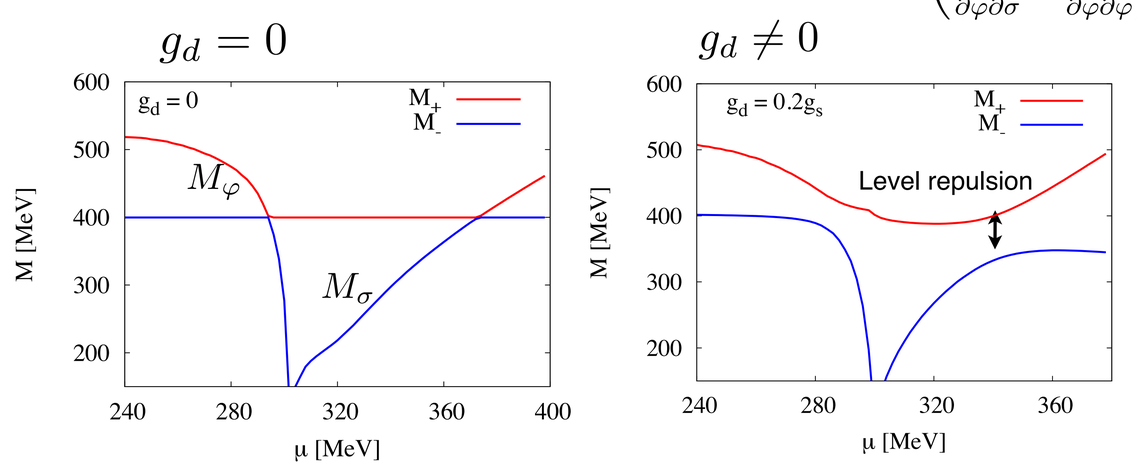
Effective potential at the critical point

$U(\sigma,\varphi)$

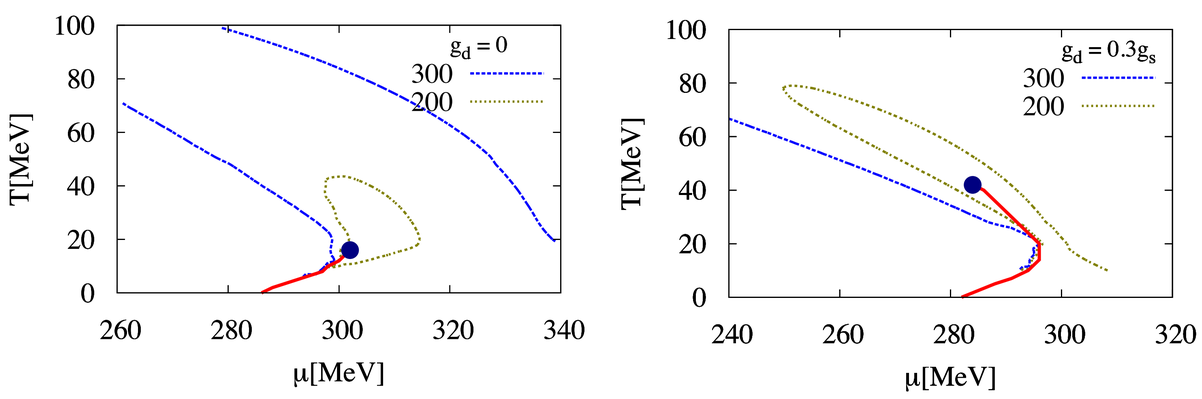


Behaviors of the soft mode mass at T = T_c

$$M = \begin{pmatrix} \frac{\partial^2 U}{\partial\sigma\partial\sigma} & \frac{\partial^2 U}{\partial\sigma\partial\varphi} \\ \frac{\partial^2 U}{\partial\varphi\partial\sigma} & \frac{\partial^2 U}{\partial\varphi\partial\varphi} \end{pmatrix}\Bigg|_{\sigma_0,\varphi_0}$$



Contour of smaller mass M_-



- A kind of level repulsion between σ and φ is founded.
- This level repulsion enhances the softening behavior of M_-.

Summary

- We have studied the QCD critical point and its critical region by using the functional-RG method.
- We have introduced a model which includes the baryon-density fluctuation mode as well as the chiral condensate.
- We have seen an expansion of the critical region with the coupling increasing.
- The expansion behavior is caused by a level repulsion of the masses. Then the inclusion of a new mode will generally cause the expansion of the critical region.