Karl-Franzens-Universität Graz
Institut für Physik, Fachbereich Theoretische Physik

Der Wissenschaftsfonds.

New developments for the fugacity expansion approach to finite density QCD

C. Gattringer, H.-P. Schadler
Karl-Franzens University of Graz

Motivation

Over the last decades lattice QCD has made large progress in studying the theory of strong-interacting quarks and gluons at finite temperature (for a recent review see, e.g., [1]) and gave deep insights in the temperature driven phases of QCD.

In the case of **finite density**, i.e., finite baryon chemical potential, the lattice QCD community has encountered a major problem which is commonly known as *"the sign problem"*. For finite baryon chemical potential the quark determinant, which is used besides the gauge action in the generation of gauge configurations as part of the Monte Carlo weight, is no longer real but complex, which does not allow for the definition as a probability and makes simulations impossible.

In this project we perform an **expansion in the fugacity parameter** $e^{\mu\beta}$ to approach small values of μ. The result is a Laurent series with canonical determinants as coefficients. It will be used to calculate the quark number density and generalized susceptibilities. We present results obtained using naive Wilson fermions and plaquette action on $8^3 \times 4$ lattices at couplings below and above the crossover.

Setup of the calculation

- **Grand canonical determinant** with **chemical potential** μ as a **fugacity series**

$$\det[D(\mu)] = \sum_{q=-q_{\max}}^{q_{\max}} e^{\mu\beta q} D^{(q)} .$$

- **Canonical determinants** $D^{(q)}$ with (fixed) quark number q

$$D^{(q)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \, e^{-iq\phi} \det[D(\mu\beta = i\phi)] .$$

- Series is exact for $q_{\max} = 6\,V$ but has to be truncated in numerical calculations.
- Calculation of the Fourier integral has to be performed numerically.
- We use naive **Wilson fermions** and apply a **dimensional reduction** [2, 3]:

$$\det[D(\mu)] = A_0 \, W ,$$

with a μ independent part A_0 and

$$W = \det[K_0 - e^{\mu\beta} K - e^{-\mu\beta} K^\dagger] .$$

Note: In this form, the matrices K_0, K are independent of μ and live on a single time slice. After building the matrices one can reuse them to calculate the determinant for any given μ, which greatly reduces the overall numerical costs.

Calculation of observables

- **Observables:** Derivatives of the partition function w.r.t. μ:

$$\chi_n^q \propto \frac{\partial^n \ln Z}{\partial(\mu\beta)^n} .$$

- **Quark number density:**

$$\frac{\chi_1^q}{T^3} = \frac{n_q}{T^3} = 2\,\frac{\beta^3}{V}\,\frac{\langle M^0 M^1\rangle_0}{\langle (M^0)^2\rangle_0} .$$

- **Quark number susceptibility:**

$$\frac{\chi_2^q}{T^2} = 2\,\frac{\beta^3}{V}\left[\frac{\langle (M^1)^2\rangle_0 + \langle M^0 M^2\rangle_0}{\langle (M^0)^2\rangle_0} - 2\left(\frac{\langle M^0 M^1\rangle_0}{\langle (M^0)^2\rangle_0}\right)^2\right] .$$

- **And higher derivatives:** χ_3^q/T and χ_4^q.
- M^n are moments of the canonical determinants

$$M^n = \sum_{q=-q_{\max}}^{q_{\max}} e^{\mu\beta q}\, q^n \, \frac{D^{(q)}}{\det[D(\mu = 0)]} .$$

Note: $\langle \ldots \rangle_0$ denotes the expectation value evaluated on configurations generated at zero chemical potential $\mu = 0$.

Quark number density and generalized susceptibilities

We calculate observables as a function of the inverse coupling $6/g^2$ for different values of μ using an ensemble of ≥ 300 configurations generated using the MILC code for an inverse mass parameter of $\kappa = 0.158$ ($m_\pi \approx 960\,$MeV).

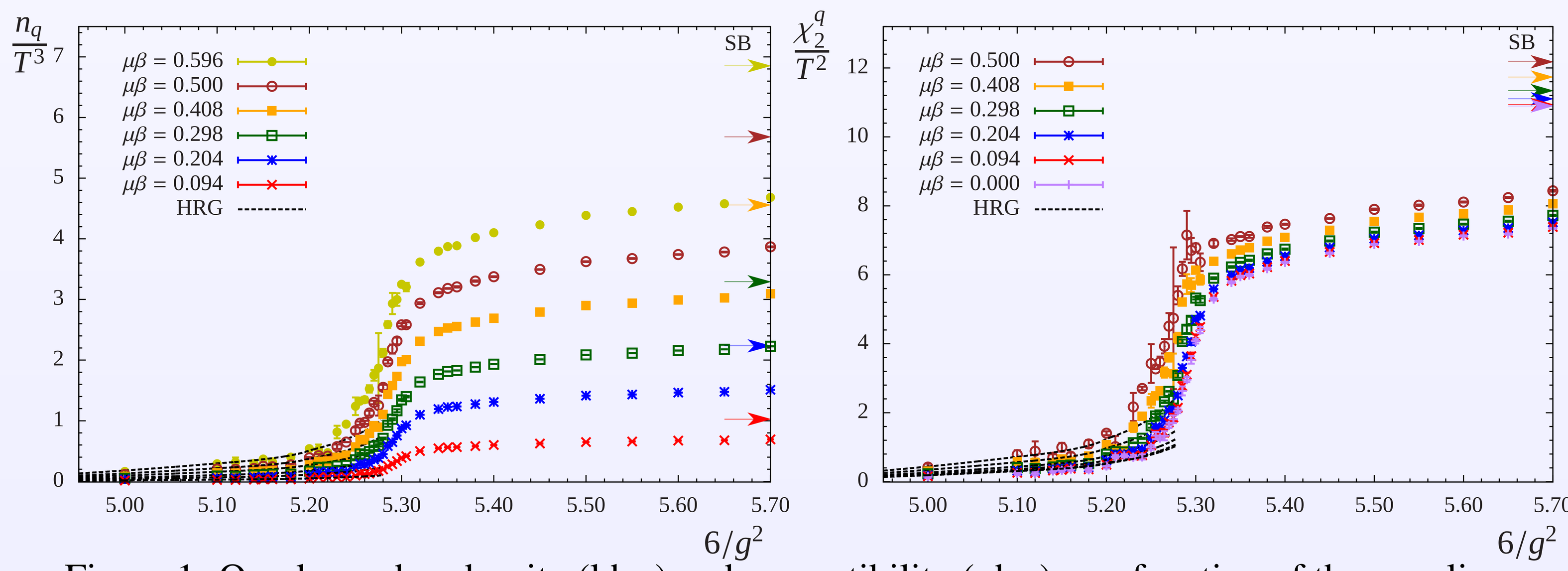

Figure 1: Quark number density (l.h.s) and susceptibility (r.h.s) as a function of the coupling.

The **quark number density and susceptibility** show a strong μ dependence above the crossover (approx. at $6/g^2 = 5.30$). In the confined phase there is good agreement with HRG results.

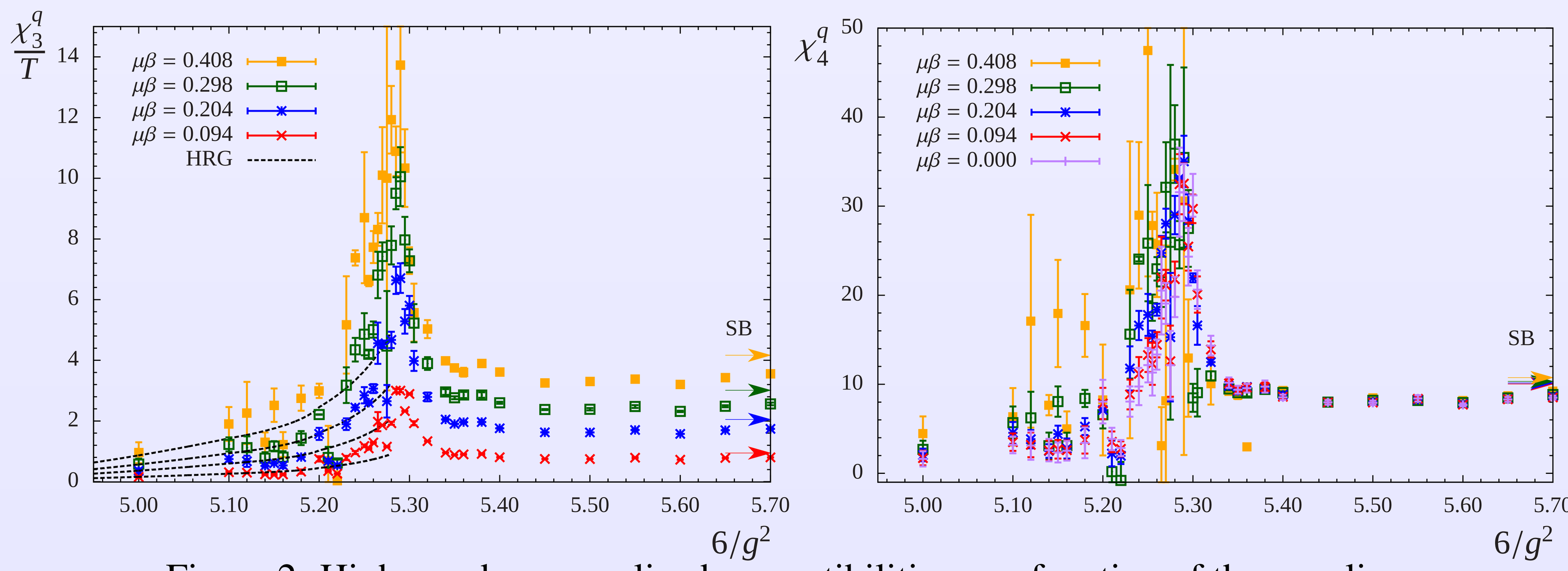

Figure 2: Higher order generalized susceptibilities as a function of the coupling.

Higher derivatives diverge at the crossover. Both, χ_3^q/T and χ_4^q, show a peak.

Connection to experiment can be established by looking at **ratios of generalized susceptibilities** [4, 5]. These quantities are related to observables which can be measured in experiments and their combinations do not depend on the volume.

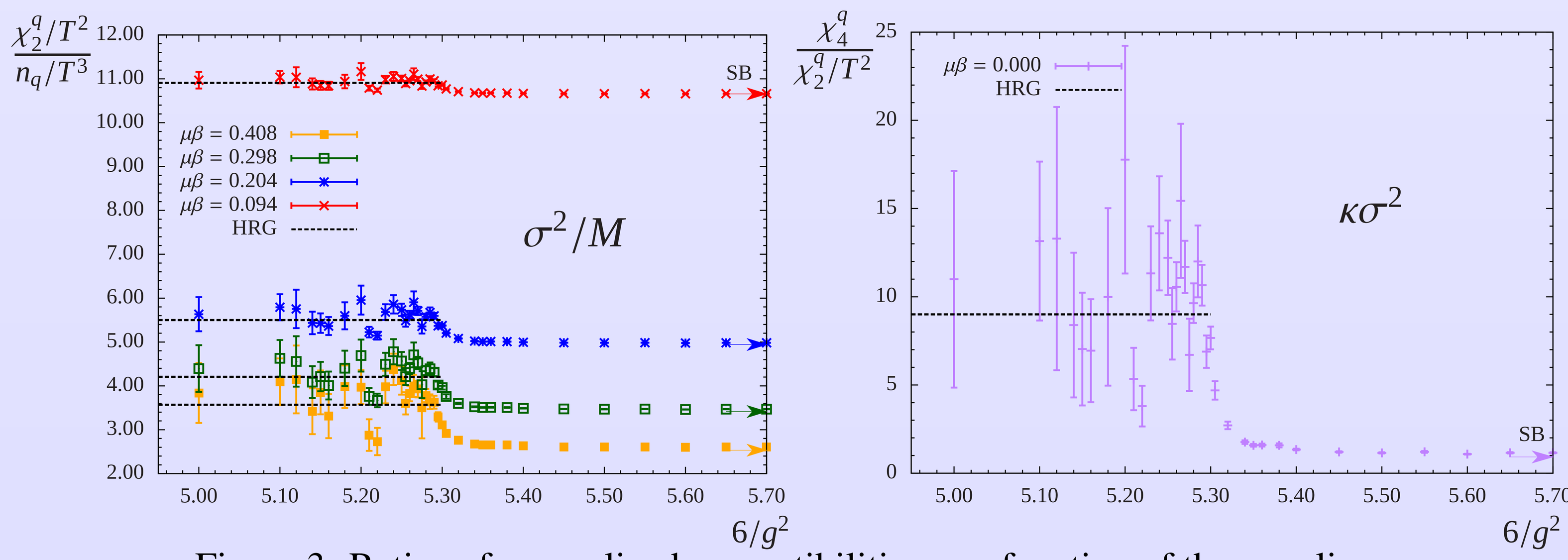

Figure 3: Ratios of generalized susceptibilities as a function of the coupling.

By looking at the ratios, both regions can be easily distinguished. Up to the crossover the results show very good agreement with HRG (no dependence on hadron spectrum!) and the deconfined region agrees very well with the SB limit.

Summary and outlook

In this project we test the fugacity expansion approach to calculate observables on the lattice at finite baryon chemical potential. Observables related to quark numbers are very well suited for this approach as they have elegant expressions in the fugacity series. We have achieved good results up to the 4th derivative w.r.t. μ and we have found good agreement with model calculations (HRG).

The generalization to other types of lattice Dirac operators is straightforward and has been tested for clover and staggered fermions. Calculations in this direction, together with improvements of the numerical techniques, larger volumes and further comparison with other approaches will be future goals.

Acknowledgments

This work is supported by the Austrian Science Fund, FWF, through the Doctoral Program "*Hadrons in Vacuum, Nuclei and Stars*" (DK W1203) and a research grant from the province of Styria.

References

[1] M. Lombardo, (2013), 1301.7324 [hep-lat].

[2] J. Danzer and C. Gattringer, Phys. Rev. D **78**, 114506 (2008).

[3] J. Danzer and C. Gattringer, Phys. Rev. D **86**, 014502 (2012).

[4] STAR Collaboration, M. M. Aggarwal *et al.*, Phys. Rev. Lett. **105**, 022302 (2010).

[5] B. Friman, F. Karsch, K. Redlich, and V. Skokov, Eur.Phys.J. **C71**, 1694 (2011), 1103.3511.