Effective lattice theory for finite temperature Yang Mills

G. Bergner[1], J. Langelage[2], O. Philipsen [1]
[1]ITP, Universität Frankfurt; [2] ITP, ETH Zürich

Introduction

Approach
- effective description of finite temperature behaviour (confined phase)
- systematic derivation from the full Yang-Mills theory

Features
- better access to certain regions in parameter space
- tested also in heavy quark region
- results for finite chemical potential possible

This work: compute/test further observables in Yang-Mills

Strong coupling effective action approach

The effective Polyakov loop action

$$e^{-S_{\text{eff}}[U_0]} = \int [dU_i] \prod_p e^{\frac{\beta}{6}\text{Tr}\left(U_p + U_p^\dagger\right)}$$

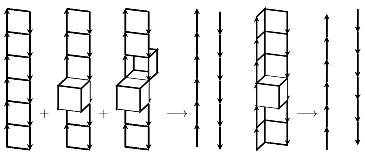
- integrating out spatial links U_i
- dimensional reduction from $3+1$D to $3D$
  $U_\mu(x,t) \to U_0(x) \to$ Polyakov lines $L(x)$
- no complete calculation possible
  $\Rightarrow$ organization of interactions in S_{eff} e.g. ordered by distance
- several approaches: inverse MC, demon methods , relative weights

$$Z = \int [dL] e^{-S_{\text{eff}}[L]}$$

Derivation of the effective action from strong coupling series
- character expansion:

$$e^{\frac{\beta}{6}\text{Tr}\left(U_p + U_p^\dagger\right)} = \sum_{r \in irreps.} (1 + d_r a_r(\beta) \chi_r(U_p))$$

- expansion parameter $u = a_f$ (resummation)
- cluster expansion



Effective action from strong coupling and simulations

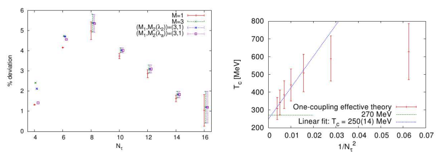
$$S_{\text{eff}} = \lambda_1 S_{\text{nearest neighbors}} + \lambda_2 S_{\text{next to nearest neighbors}} + \dots$$

- ordering principle for the interactions
  higher representations and long distances are suppressed
  (u^{N_t}; u^{2N_t}; u^{2N_t+2})
- effective couplings exponentiate:
  $\lambda_1 = u^{N_t} \exp(N_t P(u))$ (resummation)
- collect similar terms to log (resummation)

$$S_{\text{nearest neighbors}} = \sum_{<ij>} (\lambda_1 \text{Re}\, L_i L_j^* - (\lambda_1 \text{Re}\, L_i L_j^*)^2 + \dots)$$
$$= \sum_{<ij>} \log(1 + \lambda_1 \text{Re}\, L_i L_j^*)$$

Simulations of the effective theory

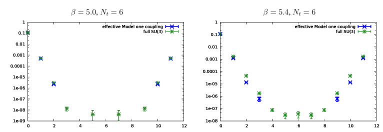
Non-perturbative effects from MC simulation of effective theory.
- as in pure SU(3) YM: 1st order phase transition, spont. broken centre symmetry
- higher representations, long distances suppressed in continuum limit
- $(\lambda_1)_c$ mapped back to $(\beta_c)_{\text{eff}} \to T_c$
- few percent difference $(\beta_c)_{\text{eff}}$ to $(\beta_c)_{\text{YM}}$



Polyakov line correlators in the effective theory

- Polyakov line correlator
  $\langle L(\vec{0}) L^\dagger(\vec{R}) \rangle$ good test for effective actions
- related to free energy in presence of heavy quarks
  $\langle L(\vec{0}) L^\dagger(\vec{R}) \rangle = \exp(-F(|\vec{R}|, T)/T)$
- continuum: depends only on $|\vec{R}|$;
  lattice: dependence on the direction
  (breaking of rotational symmetry)
- sign for the restoration of rotational symmetry in the continuum limit
- precise check

Polyakov loop correlator

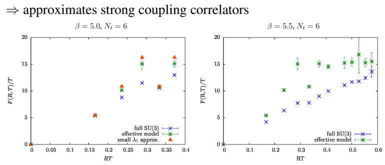


- YM strong coupling region: multilevel and multihit algorithm
- deviations close to $(\beta_c)_{\text{YM}}$; but still reasonable agreement
- larger deviations in off-axis correlator
- next to nearest neighbor interactions: small improvement

Strong coupling off-axis correlator

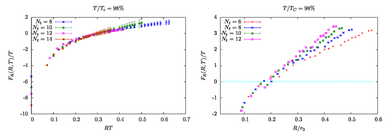
Small λ_1 behaviour: $F(R/a,T)/T = d(R/a) N_t C(\beta)$
- $d(R/a)$ smallest number of lattice spacings connecting points with distance R/a on the lattice
- breaking of rotational symmetry as in strong coupling YM
- no UV $1/r$ part at small λ_1

$\Rightarrow$ approximates strong coupling correlators



Polyakov loop correlator and continuum limit

continuum behaviour at large λ_1:
effective model close to $(\beta_c)_{\text{eff}} \leftrightarrow$ YM close to $(\beta_c)_{\text{YM}}$
- both: restoration of rotational symmetry
- YM: scaling behaviour of renormalized correlator
- effective model: still need identify scaling region ($\sqrt{\sigma}/T$)
  (using scale setting of YM)

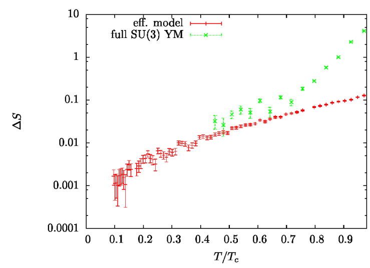


Effective theory and thermodynamics

primary observable:

$$\frac{d}{d\beta}\frac{p}{T^4} = \Delta S$$

- better agreement with YM than strong coupling expansion
- useful to get small T/T_c results



$N_\tau = 4$

Conclusions

- systematic derivation of effective PL theory:
  strong coupling series
- non-perturbative simulations of effective theory: reasonable agreement with full theory in confined phase in contrast to strong coupling results
- towards continuum limit higher orders in the expansion are important
- Can we identify intermediate scaling region?

☑ T_c from $(\lambda_1)_c$

☐ Polyakov loop correlators: must be outside perturbative region of effective theory
$\Rightarrow$ close to $(\lambda_1)_c$, below certain N_t
open issue: scale setting / renormalization in effective theory

☐ improved strong coupling results also for thermodynamics

References

[1] J. Langelage, S. Lottini and O. Philipsen, JHEP **1102** (2011) 057 [Erratum-ibid. **1107** (2011) 014] [arXiv:1010.0951 [hep-lat]].

[2] M. Fromm, J. Langelage, S. Lottini, M. Neuman and O. Philipsen, arXiv:1207.3005 [hep-lat].

[3] C. Wozar, T. Kaestner, A. Wipf and T. Heinzl, Phys. Rev. D **76** (2007) 085004 [arXiv:0704.2570 [hep-lat]].

[4] J. Greensite and K. Langfeld, arXiv:1301.4977 [hep-lat].

[5] R. De Pietri, A. Feo, E. Seiler and I. -O. Stamatescu, Phys. Rev. D **76** (2007) 114501 [arXiv:0705.3420 [hep-lat]].