Thermodynamics of dense 2-color matter

Pietro Giudice[1,2], Seamus Cotter[3], Simon Hands[1], Jon-Ivar Skullerud[3]

[1]Department of Physics, College of Science, Swansea University, United Kingdom
[2]Universität Münster, Institut für Theoretische Physik Münster, Germany
[3]Department of Mathematical Physics, National University of Ireland Maynooth, Maynooth, County Kildare, Ireland

Abstract

We study two-color QCD with two flavors of Wilson fermion as a function of quark chemical potential and temperature. We find evidence of three distinct phases at low temperature, namely a vacuum/hadronic phase, a superfluid phase, where the quark number density and diquark condensate are both very well described by a Fermi sphere of nearly-free quarks disrupted by a BCS condensate, and a deconfined phase. We present our recent results supporting this picture, focusing on the equation of state.

This presentation is based on: Phys. Rev. D **87** (2013) 034507.

Introduction

- Intensive efforts are underway to unveil the phase structure of strongly interacting matter at high density and low temperature.
- A wealth of information exists regarding possible phases and their properties in various models but no reliable, quantitative results are available yet.
- Many questions could in principle be answered by lattice QCD simulations, but unfortunately their practical feasibility is limited, due to the existence of the so called "sign problem".
- Lattice simulations may still be used to study QCD-like theories without a sign problem providing first-principles, nonperturbative results: this is the main aim of the present study.
- We focus on QCD with gauge group SU(2) (two-color QCD or QC$_2$D); it is of particular interest because it shares most of the salient features of real QCD (eg, confinement, dynamical chiral symmetry breaking and long-range interactions). It differs from QCD in that the baryons of the theory are bosons, and the lightest baryon is a pseudo-Goldstone boson, degenerate with the pion.
- We study QC$_2$D with conventional Wilson action for the gauge fields and two flavours of Wilson fermion plus a *diquark source* term (μ baryon chemical potential, $M(\mu)$ Wilson fermion matrix):

$$S_f = \bar{\psi}_1 M(\mu)\psi_1 + \bar{\psi}_2 M(\mu)\psi_2 + J\left\{\psi_2^{tr}(C\gamma_5)\tau_2\psi_1 - \bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^{tr}\right\} \ . \tag{1}$$

- It is convenient to introduce the change of variables: $\bar{\phi} = -\psi_2^{tr}C\tau_2$, $\phi = C^{-1}\tau_2\bar{\psi}_2^{tr}$ and $\psi = \psi_1$, $\bar{\psi} = \bar{\psi}_1$. Using it, we can rewrite S_f as:

$$S_f = (\bar{\psi}\bar{\phi})\begin{pmatrix} M(\mu) & J\gamma_5 \\ -\bar{J}\gamma_5 & M(-\mu)\end{pmatrix}\begin{pmatrix}\psi \\ \phi\end{pmatrix} \equiv \bar{\Psi}\mathcal{M}\Psi \ . \tag{2}$$

Expectations

At low temperature, increasing μ, we expect to see mainly three regimes:

- $\mu < \mu_o$: vacuum.
- $\mu \gtrsim \mu_o$: dilute tightly-bound quarks (weakly interacting baryons) $\to$ BEC.
- $\mu \gg \mu_o$: weakly interacting quarks Cooper pairs $\to$ BCS.

From chiral perturbation theory ($m_\pi \ll m_\rho$) we know that when $\mu \geq \mu_o \equiv \frac{1}{2}m_\pi$:

- $n_q > 0$: non zero baryon density.
- $\langle qq\rangle \neq 0$: a gauge invariant superfluid order parameter (spontaneous breaking of U(1) baryon number).

Order Parameters and Observables

- Polyakov Loop: $L = \langle \mathrm{Tr}\prod_{n_4=1}^{N_\tau} U_4(n,n_4)\rangle$ (confinement/deconfinement).
- Diquark condensate: $\langle qq\rangle = \frac{1}{V}\frac{\partial \ln Z}{\partial j}$ (superfluidity).
- Quark number density n_q.
- Quark number susceptibility:

$$\chi_q = \partial n_q/\partial\mu = \frac{T}{V_s}\left\{-\langle\left[-\bar{\Psi}\frac{\partial\mathcal{M}}{\partial\mu}\Psi\right]\rangle^2 + \langle\left[-\bar{\Psi}\frac{\partial\mathcal{M}}{\partial\mu}\Psi\right]^2\rangle + \langle\left[-\bar{\Psi}\frac{\partial^2\mathcal{M}}{\partial\mu^2}\Psi\right]\rangle\right\} \ .$$

- Pressure $p = \int_{\mu_o}^{\mu} n_q d\mu$. We use three schemes to present our results:

$$\left(\frac{p}{p_{SB}}\right)_0 = (p_{SB}^{\mathrm{cont}}(\mu))^{-1}\int_{\mu_o}^{\mu} n_q(\mu')d\mu' \ , \qquad \left(\frac{p}{p_{SB}}\right)_I = (p_{SB}^{\mathrm{lat}}(\mu))^{-1}\int_{\mu_o}^{\mu} n_q(\mu')d\mu'$$

$$\left(\frac{p}{p_{SB}}\right)_{II} = (p_{SB}^{\mathrm{cont}}(\mu))^{-1}\int_{\mu_o}^{\mu}\frac{n_{SB}^{\mathrm{cont}}}{n^{\mathrm{lat}}{}_{SB}}(\mu')n_q(\mu')d\mu' \ , \qquad \text{where} \qquad p_{SB}^{\mathrm{cont}} = \frac{N_f N_c}{12\pi^2}\left(\mu^4 + 2\pi^2\mu^2T^2 + \frac{7\pi^4}{15}T^4\right).$$

- Energy density $\varepsilon(T) = n_q\mu - \frac{1}{V}\frac{\partial Z}{\partial T^{-1}}\Big|_V$ and trace anomaly $T_{\mu\mu} \equiv \varepsilon - 3p = \frac{T}{V}\left\langle a_s\frac{\partial S}{\partial a_s}\big|_\xi\right\rangle$:

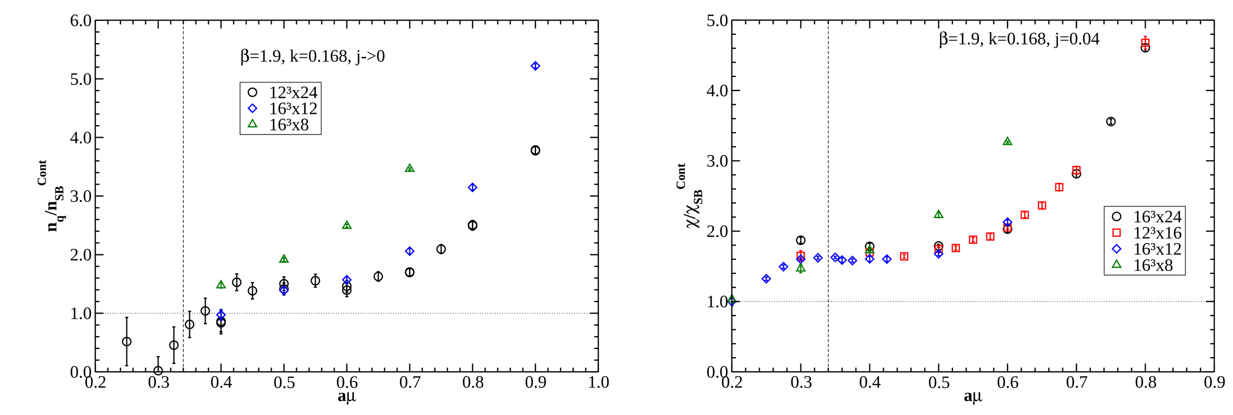
  these require knowledge of Karsch coefficients $d(\beta,\kappa,\gamma_{g,q})/d\xi|_{a_s}$ and beta functions $d(\beta,\kappa)/da_s|_{\xi=1}$ which can be determined from anisotropic simulations with $\xi = a_s/a_t$. $\gamma_{g,q}$ are bare gluon and quark anisotropies.

Results

Parameters of the simulations presented here: $\beta = 1.9$, $\kappa = 0.168$;
$m_\pi = 0.645(8)$, $m_\pi/m_\rho = 0.805(9)$, $a = 0.178(6)$ fm, $T_d(\mu = 0) = 217(23)$ MeV.

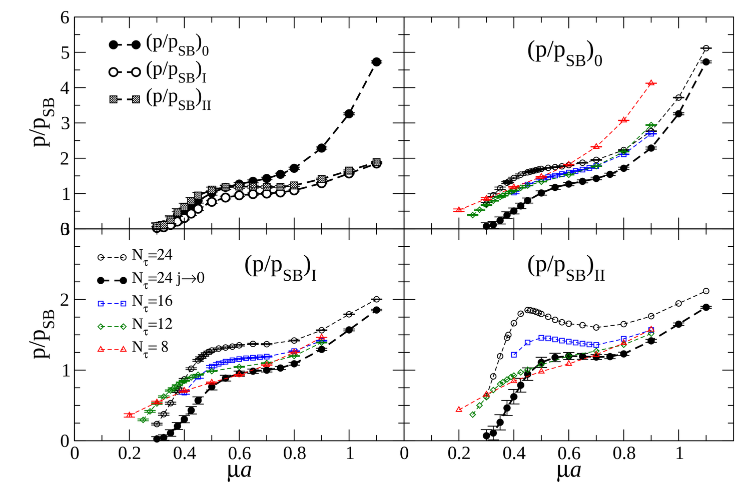
Correspondence lattice-temperature:
$N_\tau = 8 \to 132$MeV, $N_\tau = 12 \to 88$MeV, $N_\tau = 16 \to 66$MeV, $N_\tau = 24 \to 44$MeV.

Quark number density n_q and quark number susceptibility χ_q:



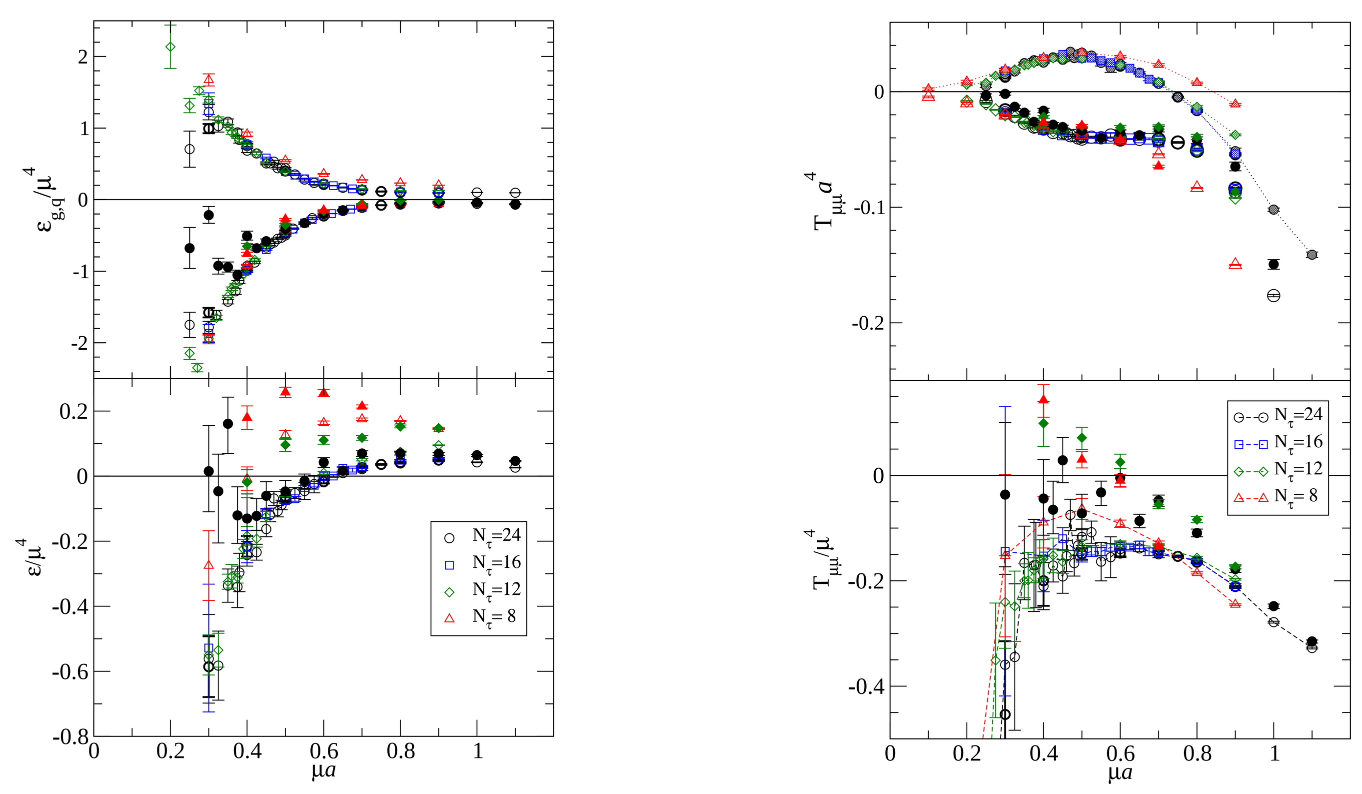
- Roughly constant in the region $0.4 \lesssim \mu a \lesssim 0.7$, i.e. approximately equal to non interacting fermions;
- The behavior of χ does not signal any abrupt release of new degrees of fredom;
- The different behaviour of the system at $N_\tau = 8$ suggests a different phase.

Pressure for our three schemes:



- Top, left: $j \to 0$, $N_\tau = 24$;
- Scheme 0: p/p_{SB} substantially exceeds unity at large μ, i.e. strong UV artefacts;
- Scheme II: presence of a bump due to IR artefacts;
- Scheme I: the coldest lattice has a plateau with value ≈ 1;
- Clearly, there is a range of μ where p scales as for free fermions.

Renormalised energy density and trace anomaly:
(Note: in these plots the additive term $n_q\mu$, present in the definition of ϵ, is not included)

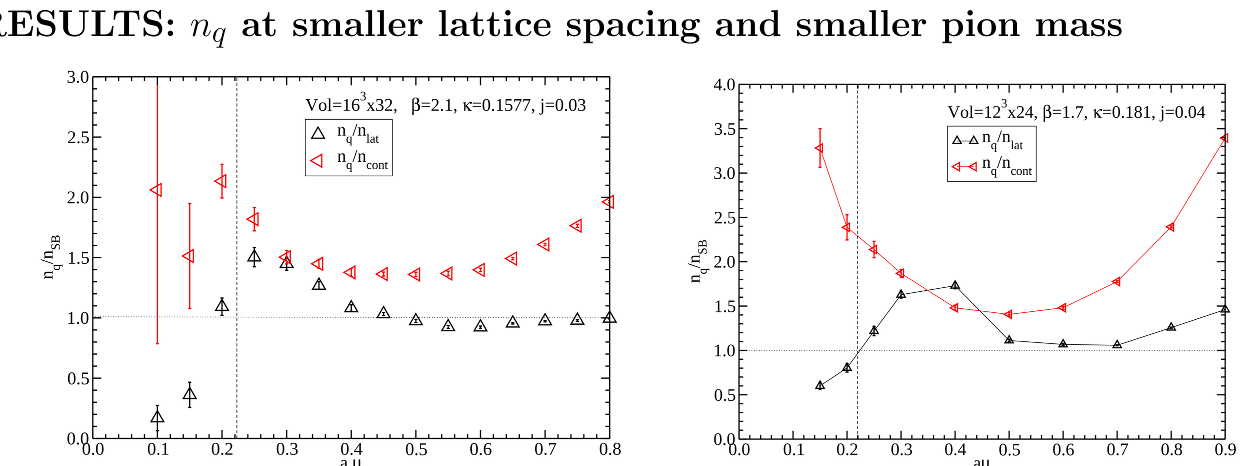


open symbols: $j = 0.04$; filled symbols: $j \to 0$
quarks: negative numbers;
gluons: positive numbers.

Top: gluon (shaded symbols, dotted lines) and quark (open symbols) at $ja = 0.04$. Filled symbols: quark contributions $j \to 0$. Bottom: $ja = 0.04$ (open symbols, dashed lines) and $j \to 0$ (filled symbols).

- ϵ_q is independent of the temperature; ϵ_g different only for the highest temperature;
- ϵ_q is sensitive to j for low μ; ϵ_g is independent of j;
- Even if ϵ_q is negative, the total one is compatible with zero at low μ and positive for higher values;
- Uncertainties in the Karsch coefficient not included: the effect may be of $\mathcal{O}(100\%)$;
- Negative trace anomaly, nearly vanishing for $0.4 \lesssim \mu a \lesssim 0.7$.

NEW RESULTS: n_q at smaller lattice spacing and smaller pion mass



- Smaller lattice spacing: $\beta = 2.1$, $\kappa = 0.1577$, $\Rightarrow m_\pi/m_\rho = 0.805(5)$, $am_\pi = 0.446(3)$, 0.122(5) fm ;
- Smaller pion mass: $\beta = 1.7$, $\kappa = 0.1810$, $\Rightarrow m_\pi/m_\rho = 0.61(5)$, $am_\pi = 0.438(15)$, 0.189(4) fm .

Conclusions/Outlook

We find evidence of three regions at low T:

1. A vacuum/hadronic phase, with $\langle qq\rangle = 0, \langle L\rangle \approx 0, \langle\bar{\psi}\psi\rangle \neq 0, n_q \approx 0$, at low T and $\mu \lesssim \mu_o = m_\pi/2$;
2. A quarkyonic phase, which is confined ($\langle L\rangle \approx 0$), chirally symmetric and characterised by SB scaling of bulk thermodynamic quantities and BCS scaling of the diquark condensate;
3. A deconfined quark–gluon plasma phase at high T (and/or large μ).

- We do not see a BEC phase (probably because the large ratio m_π/m_ρ);
- Currently we are extending our study to smaller lattice spacing and smaller m_π/m_ρ ratio.