Differences and similarities between fundamental and adjoint fermions in SU(N) gauge theories

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Aim

This work aim at clarifying what is the most essential difference between fundamental and adjoint fermions.

This subject will be talked by Dr. Kashiwa.

This poster is based on
Phys. Rev. D88 (2013) 016002
[arXiv:1304.3274].

Our question and its answer

What is the most essential difference between fundamental and adjoint quarks?

The answer is Z_3 symmetry.
Adjoint quarks preserve Z_3 symmetry,
but fundamental quarks break the symmetry.

To understand this point clearly,
we consider the new boundary condition for fundamental quarks that preserve Z_3 symmetry.

QCD action with fundamental quarks

$$S_0 = \int d^4x [\sum_f \bar{q}_f(\gamma_\nu D_\nu + m_f) q_f + \frac{1}{4g^2} F^{a\ 2}_{\mu\nu}],$$

The action is invariant under Z_3 transformation

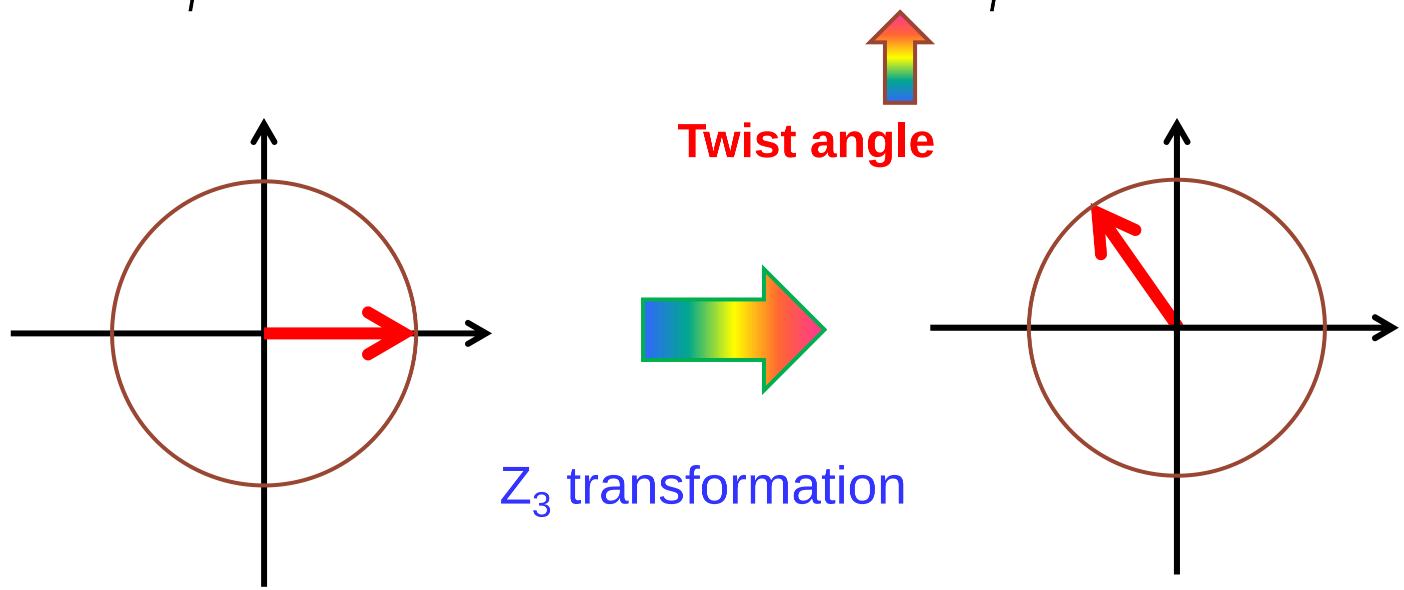
$$q \to Uq, \qquad A \to UAU^{-1} - \frac{i}{g}(\partial U)U^{-1},$$

$$U(x, \beta) = \exp[i 2\pi / 3] U(x, 0)$$

But the fermion boundary condition is not.

Boundary condition of fundamental fermion

$$q_f(x,\beta=1/T)=-\exp[i\theta]q_f(x,0)$$



$\theta=0$ $\qquad$ $\theta=2\pi/3$

Z_3 transformation rotates the twist angle by $2\pi/3$.

Construction of fundamental quarks with Z_3 symmetry

Consider degenerate three-flavor system

$$N_c = N_f = 3, \quad m_1 = m_2 = m_3$$

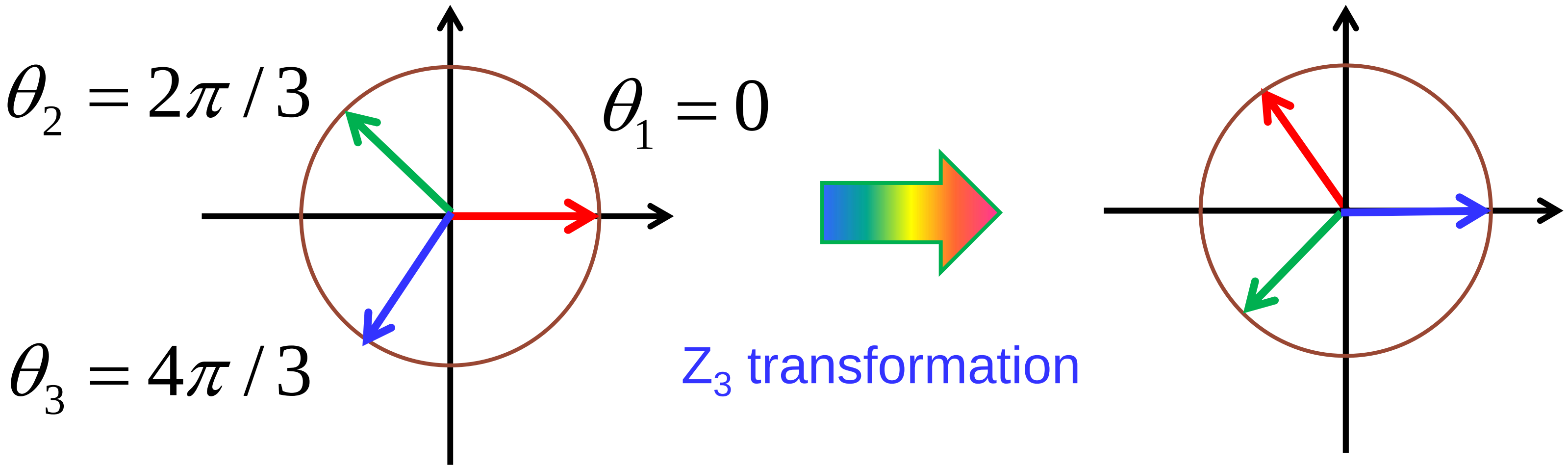
QCD with FTBC has Z_3 symmetry.

Flavor-dependent twist boundary condition (FTBC):

$$q_1(x, \beta = 1/T) = -q_1(x,0),$$
$$q_2(x, \beta = 1/T) = -\exp[i2\pi/3]q_2(x,0),$$
$$q_3(x, \beta = 1/T) = -\exp[i4\pi/3]q_3(x,0).$$

Fundamental quarks with FTBC

$$q_f(x, \beta = 1/T) = -\exp[i\theta_f]\, q_f(x,0)$$



QCD with FTBC is Z_3 invariant.

PNJL Model with FTBC

$$D_\nu = \partial_\nu + iA_\nu = \partial_\nu + i\delta_{\nu,4}A_{4,a}\tilde{\lambda}_a/2$$

PNJL Lagrangian

quark part (Nambu-Jona-Lasinio type)

$$\begin{aligned}\mathcal{L} = &\sum_f \bar{q}_f(\gamma_\nu D_\nu - \mu_f\gamma_4 + m_f)q_f \\ &- G_S \sum_f \sum_{a=0}^{8}[(\bar{q}_f\lambda_a q_f)^2 + (\bar{q}_f i\gamma_5\lambda_a q_f)^2] \\ &+ G_D\left[\det_{ij}\bar{q}_i(1+\gamma_5)q_j + \det_{ij}\bar{q}_i(1-\gamma_5)q_j\right] \\ &+ \mathcal{U}(\Phi[A], \bar{\Phi}[A], T),\end{aligned}$$

gluon potential

$$\Phi = \frac{1}{3}\mathrm{Tr}_c e^{-iA_4/T}$$

FTBC

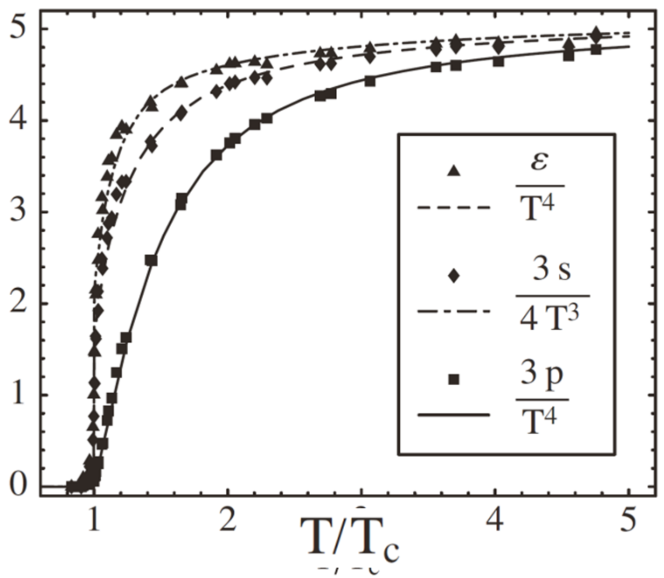
$$q_f(x, \beta = 1/T) = -\exp[i\theta_f]q_f(x, 0)$$

Polyakov potential

$$\mathcal{U} = T^4\Big[-\frac{a(T)}{2}\Phi^*\Phi + b(T)\ln(1-6\Phi\Phi^* + 4(\Phi^3+\Phi^{*3}) - 3(\Phi\Phi^*)^2)\Big],$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3.$$

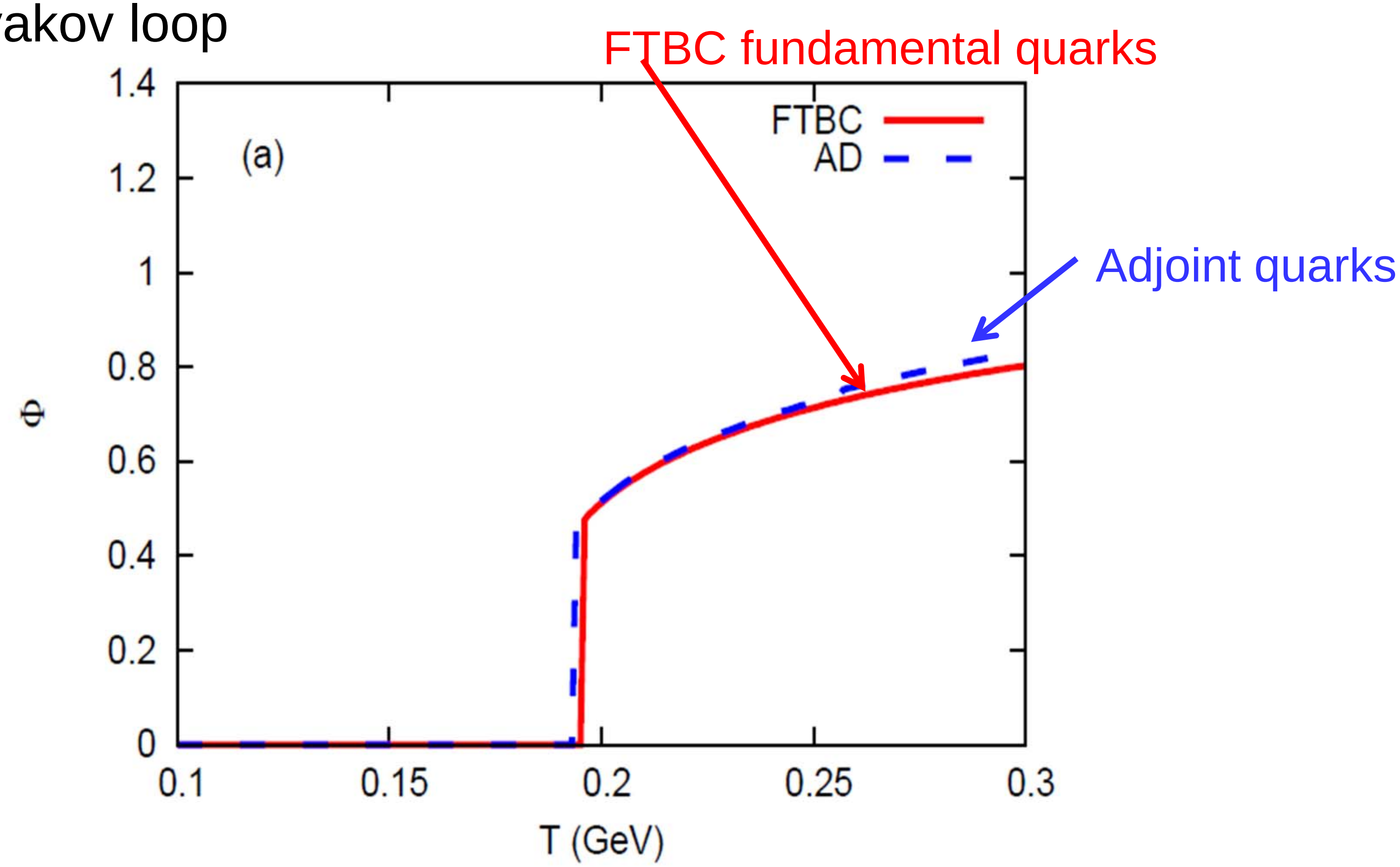
Rößner Ratti, Weise; PRD**75**



It reproduces the lattice data in the pure gauge limit.

PNJL with FTBC and adjoint quarks

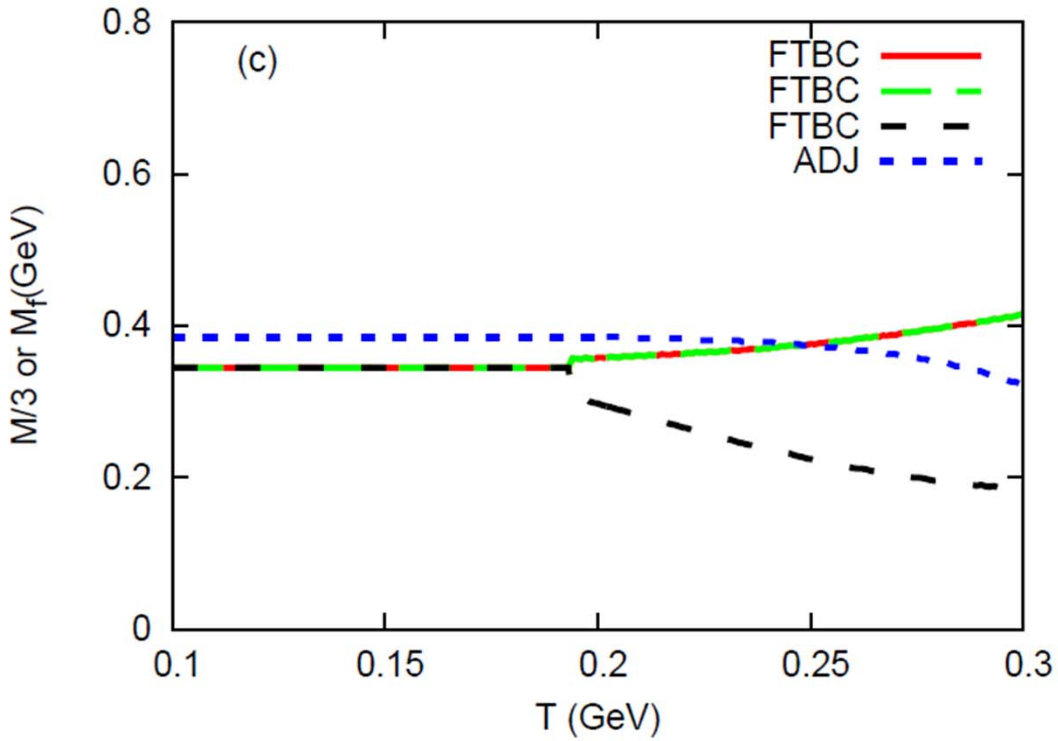
Polyakov loop



FTBC and adjoint quarks preserve Z_3 symmetry
and hence the results are similar to each other for the Polyakov loop.

PNJL with FTBC and adjoint quarks

Dynamical quark mass (chiral condensate)

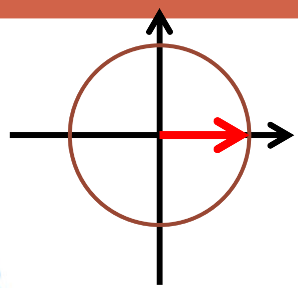


FTBC and adjoint quarks yield similar results also for chiral condensate at T smaller than the critical temperature.

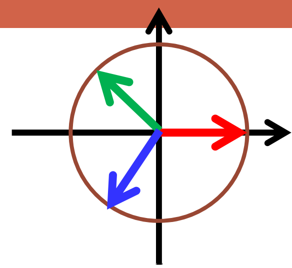
PNJL with the standard BC and FTBC

From Phys.Lett. B718 (2012) 130-135

Standard BC

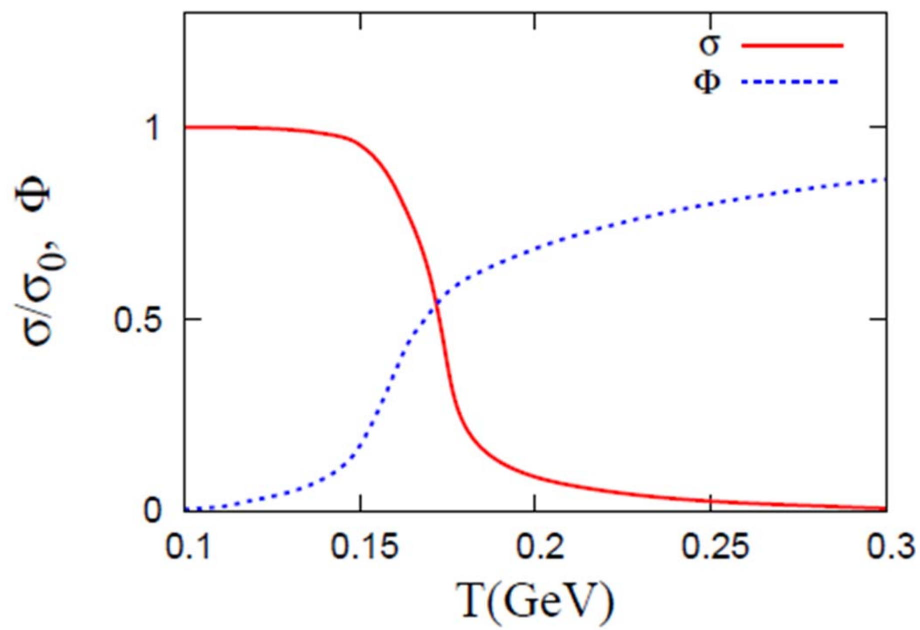


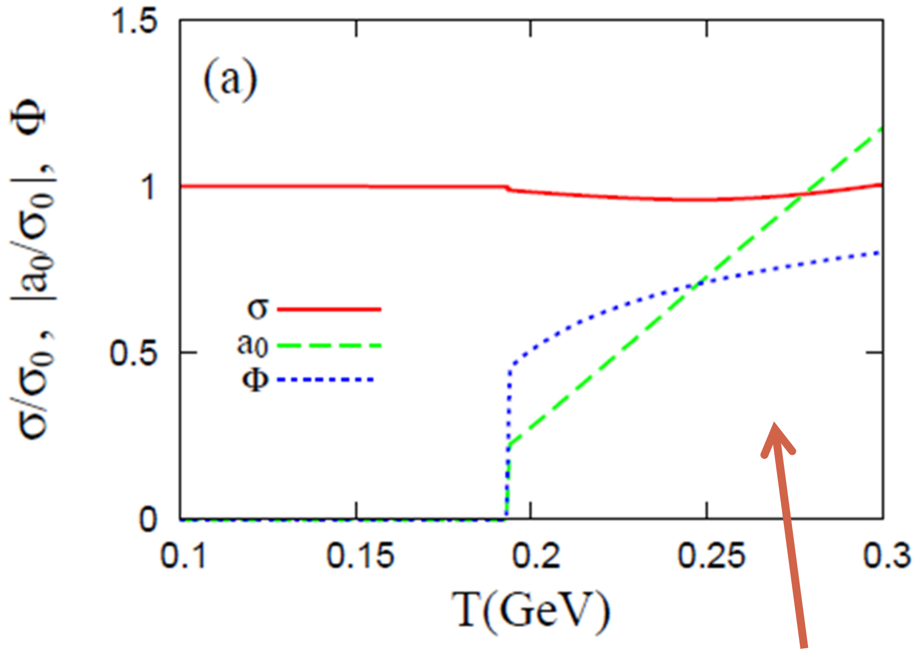
FTBC



$$\sigma \equiv (\sigma_u + \sigma_d + \sigma_s)/3$$

$$a_0 \equiv \sigma_u - \sigma_d = \sigma_u - \sigma_s$$





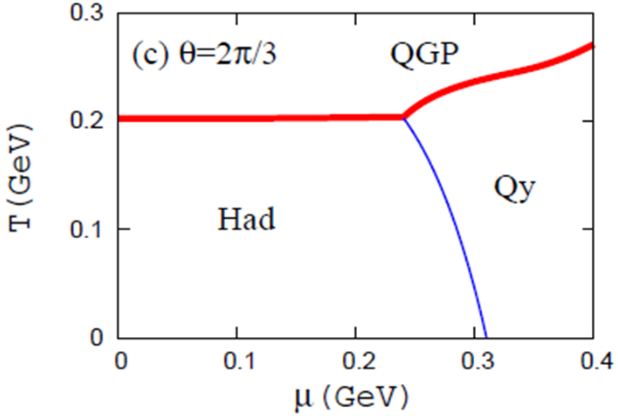
The two BCs yield different results.

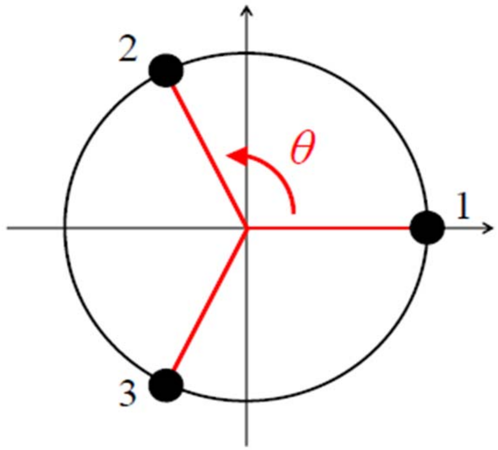
SSB of Z_3 symmetry

Phase diagram

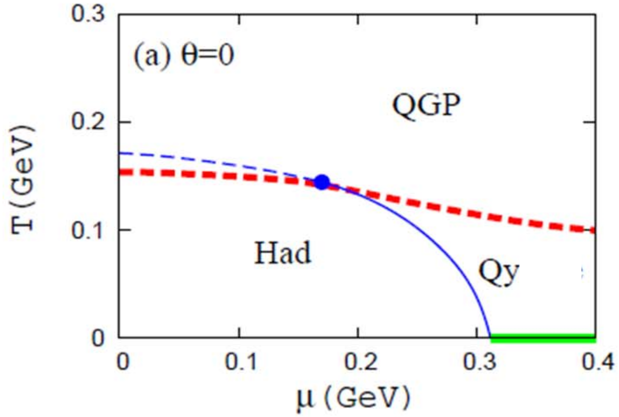
From Phys.Lett. B718 (2012) 130-135

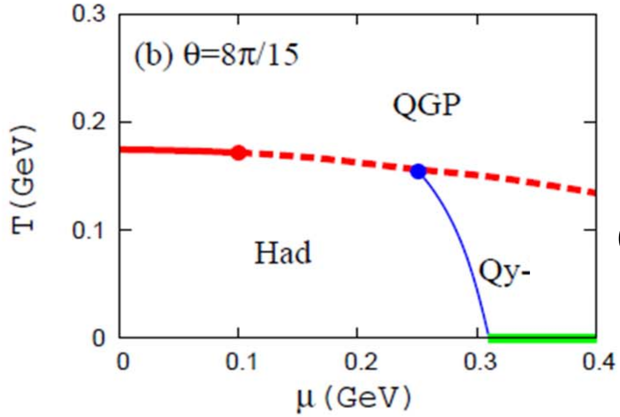
PNJL with FTBC





PNJL with standard BC





quarkyonic phase

Summary

1. FTBC fundamental and adjoint quarks yield similar results, since the two fermions preserve Z_3 symmetry.
2. Fundamental and FTBC quarks yield rather different results.
3. Therefore, the essential difference between fundamental and adjoint quarks comes from the presence or absence of Z_3 symmetry.

Collaborators

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